Differentiating Eq. (5–53) with respect to ξ and consistent with the previous developments of the two-equation locally nonsimilar boundary layer model, neglecting the term $2\xi(f'S'_\xi - S_\xi f'')$ yields the first auxiliary momentum equation, i.e.

$$S''' + fS'' - 2\beta f'S' + f''S + \frac{d\beta}{d\xi}\left(1 - f'^2\right) = 2(f'S' - f''S)$$
$$+ 2\xi(S'^2 - S''S) + 2\xi\alpha_4(1-\theta)\left(1 + \frac{1}{\alpha_4}\frac{d\alpha_4}{d\xi}\right) - 2\xi\alpha_4 t \qquad (5-54)$$

The energy equation and the first auxiliary energy equation are identical to those for the two-equation locally nonsimilar forced convection boundary layer model, i.e., Eqs. (4–124) and (4–125). For completeness they are repeated here

$$\theta'' + \mathrm{Pr}\, f\theta' = \mathrm{E}_1\mathrm{Pr}\, f''^2 + 2\xi\mathrm{Pr}\,(S\theta' - tf')$$
$$+ 2\xi\mathrm{Pr}\left[\alpha_1(1-\theta) + \alpha_2\theta\right]f' \qquad (4-124)$$

and $$t'' + \mathrm{Pr}\,(S\theta' + ft') = \mathrm{Pr}\left(\frac{d\mathrm{E}_1}{d\xi}f''^2 + 2\mathrm{E}_1 f''S''\right)$$
$$+ 2\mathrm{Pr}\left[(S\theta' - tf') + \alpha_1(1-\theta)f' + \alpha_2\theta f'\right]$$
$$+ 2\xi\mathrm{Pr}\left\{(St' - S't) + \alpha_1\left[(1-\theta)S' - tf'\right]\right.$$
$$\left. + \alpha_2(tf' + \theta S') + \frac{d\alpha_1}{d\xi}(1-\theta)f' + \frac{d\alpha_2}{d\xi}\theta f'\right\}$$
$$(4-125)$$

The appropriate boundary conditions, including the effects of mass transfer at the surface, are given by Eqs. (3–106b, c), (3–127), (3–118), (3–132), (4–122a, b) and (4–127a, b). In particular, at the surface the boundary conditions are

$$f(\xi,0) + 2\xi S(\xi,0) = -\frac{v(x)}{U(x)}\left(\frac{2\ell U_\infty \xi}{\nu}\right)^{1/2} \qquad (3-127)$$
$$f'(\xi,0) = 0 \qquad S'(\xi,0) = 0 \qquad (3-118b, d)$$
$$S(\xi,0) = -\frac{1}{3}(\mathrm{Re}_\ell)^{1/2}(2\xi)^{-1/2}\frac{v_0}{U_\infty} \qquad (3-132)$$
$$\theta(\xi,0) = 0 \qquad t(\xi,0) = 0 \qquad (4-122a, 4-127a)$$

and as $\eta \to \infty$

$$f'(\xi,\eta\to\infty) \to 1 \qquad S'(\xi,\eta\to\infty) \to 0 \qquad (3-118g, h)$$
$$\theta(\xi,\eta\to\infty) \to 1 \qquad t(\xi,\eta\to\infty) \to 0 \qquad (4-122b, 4-127b)$$