Differentiating Eq. (5–53) with respect to ξ and consistent with the previous developments of the two-equation locally nonsimilar boundary layer model, neglecting the term $2\xi(f'S'_\xi - S_\xi f'')$ yields the first auxiliary momentum equation, i.e.

$$S''' + fS'' - 2\beta f'S' + f''S + \frac{d\beta}{d\xi}\left(1 - f'^2\right) = 2(f'S' - f''S)$$
$$+ 2\xi(S'^2 - S''S) + 2\xi\alpha_4(1-\theta)\left(1 + \frac{1}{\alpha_4}\frac{d\alpha_4}{d\xi}\right) - 2\xi\alpha_4 t \qquad (5-54)$$

The energy equation and the first auxiliary energy equation are identical to those for the two-equation locally nonsimilar forced convection boundary layer model, i.e., Eqs. (4–124) and (4–125). For completeness they are repeated here

$$\theta'' + \mathrm{Pr}\, f\theta' = \mathrm{E_1 Pr}\, f''^2 + 2\xi \mathrm{Pr}\,(S\theta' - tf')$$
$$+ 2\xi\mathrm{Pr}\left[\alpha_1(1-\theta) + \alpha_2\theta\right]f' \qquad (4-124)$$

and

$$t'' + \mathrm{Pr}\,(S\theta' + ft') = \mathrm{Pr}\left(\frac{d\mathrm{E_1}}{d\xi}f''^2 + 2\mathrm{E_1}f''S''\right)$$
$$+ 2\mathrm{Pr}\left[(S\theta' - tf') + \alpha_1(1-\theta)f' + \alpha_2\theta f'\right]$$
$$+ 2\xi\mathrm{Pr}\Big\{(St' - S't) + \alpha_1\left[(1-\theta)S' - tf'\right]$$
$$+ \alpha_2(tf' + \theta S') + \frac{d\alpha_1}{d\xi}(1-\theta)f' + \frac{d\alpha_2}{d\xi}\theta f'\Big\}$$
$$(4-125)$$

The appropriate boundary conditions, including the effects of mass transfer at the surface, are given by Eqs. (3–106*b, c*), (3–127), (3–118), (3–132), (4–122*a, b*) and (4–127*a, b*). In particular, at the surface the boundary conditions are

$$f(\xi,0) + 2\xi S(\xi,0) = -\frac{v(x)}{U(x)}\left(\frac{2\ell U_\infty \xi}{\nu}\right)^{1/2} \qquad (3-127)$$

$$f'(\xi,0) = 0 \qquad S'(\xi,0) = 0 \qquad (3-118b,d)$$

$$S(\xi,0) = -\frac{1}{3}(\mathrm{Re}_\ell)^{1/2}(2\xi)^{-1/2}\frac{v_0}{U_\infty} \qquad (3-132)$$

$$\theta(\xi,0) = 0 \qquad t(\xi,0) = 0 \qquad (4-122a, 4-127a)$$

and as $\eta \to \infty$

$$f'(\xi,\eta\to\infty) \to 1 \qquad S'(\xi,\eta\to\infty)\to 0 \qquad (3-118g,h)$$

$$\theta(\xi,\eta\to\infty)\to 1 \qquad t(\xi,\eta\to\infty)\to 0 \qquad (4-122b, 4-127b)$$