Altitude Effects

Part 1

David F. Rogers

In our previous discussion of the effects of lowering the gear and flaps we found that the rate-of-climb and velocity for maximum rate-of-climb were both significantly reduced. Further we found that the range of velocities that gave positive rates-of-climb was also reduced. Now we want to take a look at the effects of altitude on both the power required to maintain level flight and on the thrust power available. We need this information so that next time we can look at the effect of altitude on the rate-of-climb.

Recalling the equation for the power required to maintain level flight (by now you should be very familiar with this equation)

$$P_r = \underbrace{\frac{\sigma\rho_{\rm SL}}{2} f V^3}_{\text{parasite}} + \underbrace{\frac{2}{\sigma\rho_{\rm SL}}\frac{1}{\pi e}\left(\frac{W}{b}\right)^2 \frac{1}{V}}_{\text{effective induced}}$$

where σ (sigma) is the ratio of the density at altitude to that at sea level, $\sigma = \rho/\rho_{\rm SL}$, on a standard day.

Since we are concerned only with density effects, we can simplfy this equation to

$$P_r = \underbrace{\text{Constant}\,\sigma}_{\text{parasite}} + \underbrace{\frac{\text{Konstant}}{\sigma}}_{\text{effective induced}}$$

where Constant and Konstant are constants.

Here we see that since the density at altitude is less than the density at sea level that the density ratio σ is less than one for any altitude above sea level. This means that the parasite power required *decreases* as the altitude *increases* but the effective induced power required *increases* as the altitude increases. The right side of the power required curve depends mainly on the parasite power required and the left side on the effective induced power required (remember ground school). You can see this in Figure 1 where power required curves for sea level, 5, 10, 15 and 20,000 feet are given. The appropriate altitudes for the power required curves are shown on the left side of the figure.

Now let's look at the effect of altitude on the minimum power required for steady level flight. Recall that the full equation is

$$P_{r_{\rm min}} = 2.48\,\frac{f^{1/4}}{\sqrt{\sigma\rho_{\rm SL}}}\left(\frac{1}{\pi e}\right)^{3/4}\left(\frac{W}{b}\right)^{3/2} = \frac{\text{Constant}}{\sqrt{\sigma}}$$

but again simplifies to the result shown on the right when we consider only density effects.

Again, because the density ratio, σ, occurs in the denominator (the bottom) of the equation and is less than one, the minimum power required for steady level flight increases with altitude. A look at Figure 1 confirms this. For example, at sea level the minimum power required is approximately 81 horsepower while at 15,000 feet it is approximately 107 horsepower.

Finally let's look at the effect of altitude on the velocity (TAS) for minimun power required for steady level flight. Recall the full equation

$$V_{P_{r_{\rm min}}} = \left(\frac{4}{3\pi f e}\right)^{1/4}\sqrt{\frac{1}{\sigma\rho_{\rm SL}}}\sqrt{\frac{W}{b}} = \frac{\text{Constant}}{\sqrt{\sigma}}$$

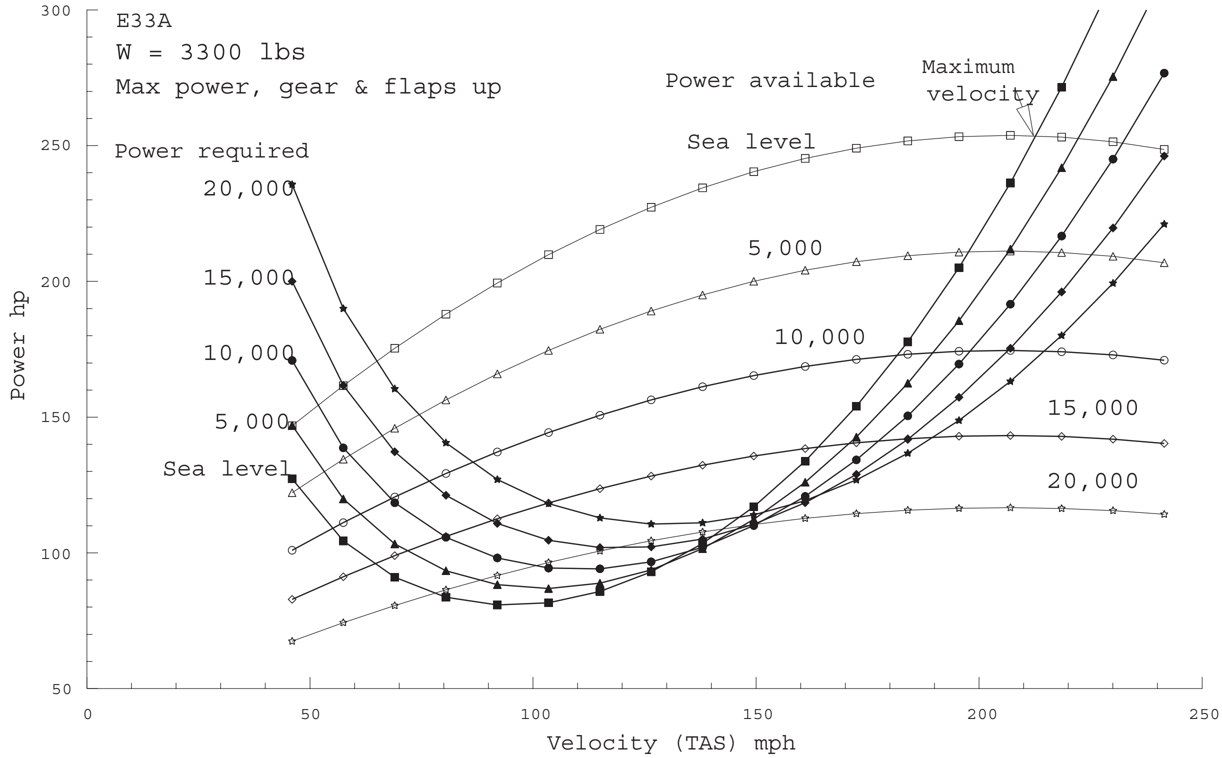

Figure 1. Power required and available vs velocity.

which, because we are only considering density effects, simplifies to the result on the right.

Again, because the density ratio, σ, occurs in the denominator and is less than one, the velocity for minimum power required for steady level flight also increases with altitude as shown in Figure 1. For example, at sea level the velocity (TAS) for minimum power required is approximately 95 mph while at 15,000 feet it is approximately 126 mph.

Also shown on Figure 1 are maximum thrust power available curves for the same altitudes, i.e. at full throttle and 2700 rpm for a model 33A. From experience we all know that for a normally aspirated piston engine, as in most Bonanzas, the power available decreases with increasing altitude. (However, this is not true for turbine powered aircraft.) But, how do we determine this? One way is to assume that the engine power available at a specific altitude depends on the pressure at that altitude. We can write this as

$$P_{a_{alt}} = \delta P_{a_{\mathrm{SL}}}$$

where δ is the ratio of pressure at a specific altitude to that at sea level. A small table giving the density and pressure ratios for various altitudes is appears below.

To find the maximum power available at a particular altitude simply multiple the maximum power at sea level, e.g., 285 for a model 33A, by the value for δ given in the table. For example, at an altitude of 10,000 feet $\delta = 0.6877$ and the maximum power available is $285 \times 0.6877 = 200$ hp.

Looking at Figure 1 notice that for each altitude, except sea level and 20,000 feet, the power available and power required curves intersect or cross in two places. The power required curve and the power available curve for the same altitude are indicated by the same shape filled and open symbols respectively. Since in steady level flight the power required must equal the power available, the intersection of the curves gives the velocities at which the airplane will fly at that power setting

Altitude feet	Density ratio σ	Pressure ratio δ
sea level	1.0000	1.0000
4000	0.8881	0.8637
5000	0.8617	0.8320
6000	0.8357	0.8014
7000	0.8106	0.7716
8000	0.7860	0.7428
9000	0.7620	0.7148
10000	0.7385	0.6877
12000	0.6360	0.6823
15000	0.6292	0.5643
20000	0.5328	0.4595

and that altitude. For any other velocity the airplane will either descend or climb. We all know this from experience. For example, if we are stabilized in cruise at a particular velocity, and *without changing the power setting*, decrease the velocity by pulling back on the yoke or increasing the nose up trim, the airplane starts to climb. It will continue to climb until the power available equals the power required at some new higher altitude but at a lower velocity (TAS). Go out and try it. I'd suggest using the trim and not changing it much. It takes a while to restabilize so be patient. Also you don't need to use maximum power, it works just as well at partial powers, try say 65% at 8,500 feet.

The right hand intersection of the curves gives the maximum velocity in level flight at that altitude. First, notice that for a normally aspirated piston engine the maximum velocity in level flight occurs at sea level. Figure 1 gives a maximum velocity of approximately 212 mph which is close to the 208 mph given by Larry Ball in his book *Those Incomparable Bonanzas*. At 15,000 feet the maximum velocity (TAS) decreases to approximately 183 mph.

What about at 20,000 feet? The curves do not intersect! This means that the airplane does not have enough power available to maintain steady level flight at 20,000 feet. The Beech POH (old version) shows an absolute ceiling, defined as the altitude at which the rate-of-climb is zero, of approximately 19,800 feet which, along with our result for the maximum velocity at sea level, increases our confidence in the results.

The lefthand intersection gives the minimum velocity (TAS) at which the aircraft can maintain steady level flight. Notice that at high altitudes the minimum velocity for steady level flight is *greater* than the power on stall velocity. For example, at 15,000 feet the minimum velocity is approximately 92 mph which is greater than the power on stall velocity (TAS) of 72 mph at 15,000 feet.

From looking at Figure 1 and the above discussion you can see that as the altitude increases the velocity envelope — minimum to maximum velocity in steady level flight — narrows. This makes speed control particularly important at high altitudes. At the absolute altitude the aircraft can maintain steady level flight at only *one* velocity. For a constant propeller efficiency, that velocity is the velocity for minimum power required. Because, in practice, propeller efficiency varies with velocity, the actual velocity is a bit higher than the velocity for minimum power required. The velocity for minimum power required is not given in the POH. However, as a first approximation you can use the velocity for an obstacle clearance take-off.

Any time the power required to maintain steady level flight exceeds the power available the aircraft must descend. At high velocities, i.e., to the right of the point where the curves intersect, the velocity in the descent exceeds the maximum steady level flight velocity for that altitude. At low velocities, i.e., to the left of the point where the curves intersect, the velocity in the descent is less than the minimum steady level flight velocity. The rate-of-descent at these velocities is calculated by

$$\text{Rate-of-descent} = \frac{\text{power available} - \text{power required}}{\text{weight}}$$

which comes out negative since the thrust power available is less than the power required.

One last detail needs explaining. Notice that the power required and power available curves do not intersect at low velocities. Fundamentally, that is because the aircraft stalls first! However, it also means that you can fly the aircraft in steady level flight just above the stall velocity and even have a small positive rate-of-climb! We have all experienced this when practicing slow flight.

Next time we'll look at the effects of altitude on the absolute ceiling with gear and flaps down. This affects our ability to go around at high altitude airports.