Some Comments On Angle of Attack Systems Calibration

by

David F. Rogers, PhD, ATP
Professor of Aerospace Engineering (Emeritus)
Annapolis, MD 21409
410-271-1968

Let's start with some rather simple mathematics.

First we ask the simple question: Is there one angle of attack that is fundamental to the aircraft, i.e., is designed into the aircraft? The answer is yes. That angle of attack is the angle of attack for maximum lift to drag ratio otherwise known at best glide or range speed. Let's show that. From the expression for the lift coefficient we have

$$C_L = a\alpha = \frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2} \quad \text{or} \quad \alpha = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2} \tag{1}$$

where α is the aircraft absolute angle of attack, n is the load factor, σ is the ratio of the density at altitude to that at sea level on a standard day, ρ_{SSL} is the density at sea level on a standard day, W is the aircraft weight, S is the aircraft wing area, V is the true airspeed (TAS) and a is the aircraft linear lift curve slope. Rewriting Eq(1) specifically for the speed for maximum lift to drag ratio, $V_{L/D\text{max}}$ yields

$$\alpha_{L/D\text{max}} = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2_{L/D\text{max}}} \tag{2}$$

The speed for L/D_{max} can be shown to be

$$V_{L/D\text{max}} = \left(\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}\right)^{1/2} \quad \text{or} \quad V^2_{L/D\text{max}} = \frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}} \tag{3}$$

where b is the wing span, f is the equivalent parasite drag and e is the Oswald aircraft efficiency factor.

Substituting $V^2_{L/D\text{max}}$ from Eq(3) into Eq(2) yields

$$\alpha_{L/D\text{max}} = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S} \quad \frac{\sigma\rho_{\text{SSL}}}{2n}\frac{b}{W}\frac{\sqrt{\pi f e}}{1} \quad \text{or} \quad \alpha_{L/D\text{max}} = \frac{1}{a}\frac{\cancel{2n}}{\cancel{\sigma\rho_{\text{SSL}}}}\frac{\cancel{W}}{S} \quad \frac{\cancel{\sigma\rho_{\text{SSL}}}}{\cancel{2n}}\frac{b}{\cancel{W}}\frac{\sqrt{\pi f e}}{1} \tag{4}$$

or

$$\alpha_{L/D\text{max}} = \frac{1}{a}\frac{b}{S}\sqrt{\pi f e} \tag{5}$$

Which does *not* depend of weight (W), load factor (n) or density altitude (σ) but *only* on aircraft *design* parameters wing span (b), wing area (S), parasite drag (f) and Oswald efficiency factor (e). Hence, the angle of attack for maximum lift-to-drag ratio is a fundamental angle of attack of the *aircraft*.

There are two other angles of attack that are simple numerical multiples of the absolute angle of attack for maximum lift to drag ratio. These are the absolute angle of attack for minimum power required, $\alpha_{P_{R_{\text{min}}}}$, (maximum endurance and minimum sink rate) and the absolute angle of attack for Carson cruise, α_{CC}

$$\alpha_{P_{R_{\text{min}}}} = \sqrt{3}\,\alpha_{L/D\text{max}} = 1.73\,\alpha_{L/D\text{max}} \tag{7}$$

and

$$\alpha_{\text{CC}} = \frac{1}{\sqrt{3}}\,\alpha_{L/D\text{max}} = 0.58\,\alpha_{L/D\text{max}} \tag{8}$$

These angles of attack are also *independent* of density altitude, weight and load factor and hence are fundamental angles of attack of the *aircraft.*

The velocities for $V_{P_{R_{\min}}}$ and V_{CC} are also simple numerical multiples of the $V_{L/D\max}$. Specifically,

$$V_{P_{R_{\min}}} = \frac{1}{\sqrt[4]{3}} V_{L/D\max} = 0.76 V_{L/D\max} \tag{9}$$

$$V_{\mathrm{CC}} = \sqrt[4]{3}\, V_{L/D\max} = 1.32 V_{L/D\max} \tag{10}$$

However, the velocity (TAS) for $V_{L/D\max}$ does depend on weight, density altitude and load factor as shown in Eq(3). Hence, $V_{P_{R_{\min}}}$ and V_{CC} also depend on weight, density altitude and load factor.

Recalling that equivalent airspeed, EAS, which, for a typical general aviation piston-propeller aircraft, is the same as calibrated airspeed, CAS

$$EAS = \sqrt{\sigma} V = \sqrt{\sigma} TAS \tag{11}$$

removes the dependence on density altitude. Furthermore, for any steady wings level flight condition, e.g., steady level flight, steady climbing flight or steady descending flight, the load factor, $n = 1$. Hence, the load factor, n, is effectively removed from the equation. Specifically Eq(3) is now,

$$\mathrm{EAS}_{L/D\max} = \left(\frac{2}{\rho_{\mathrm{SSL}}} \frac{W}{b} \frac{1}{\sqrt{\pi f e}} \right)^{1/2} \tag{12}$$

and

$$EAS_{P_{R_{\min}}} = \frac{1}{\sqrt[4]{3}} \mathrm{EAS}_{L/D\max} = 0.76 \mathrm{EAS}_{L/D\max} \tag{13}$$

and

$$EAS_{\mathrm{CC}} = \sqrt[4]{3}\, \mathrm{EAS}_{L/D\max} = 1.32\, \mathrm{EAS}_{L/D\max} \tag{14}$$

From Eq(12) we see that given the equivalent airspeed for maximum lift to drag ratio, $\mathrm{EAS}_{L/D\max}$, for any weight, e.g., aircraft gross weight, that the equivalent airspeed for any other weight is simply

$$\frac{\mathrm{EAS}_{L/D\max}|_2}{\mathrm{EAS}_{L/D\max}|_1} = \sqrt{\frac{W_{L/D_{max}}\big|_2}{W_{L/D_{max}}\big|_1}} \tag{15}$$

The equivalent airspeed for $EAS_{P_{R_{\min}}}$ and EAS_{CC} are adjusted similarly.

What does all this mathematics have to do with calibrating a pressure based angle-of-attack data acquisition system? The answer is a lot.

From the discussion above of angle of attack and its independence with respect to weight, density altitude and load factor we can conclude that angle of attack, in general, is independent of weight, density altitude and load factor as far as calibrating a pressure based angle-of-attack data acquisition system is concerned. However, the primary instrument, i.e., the the airspeed indicator, used by the pilot to select angle of attack calibration data points is not.

In fact, fundamentally it does not matter what indicated airspeeds the pilot chooses provided that the airspeed indicator itself is reasonably calibrated, the correction from indicated airspeed to calibrated airspeed is reasonably small and that data points are adequately spaced with respect to angle of attack.

It is also required that the calibration 'curve' be properly normalized using data from within the *same* pressure field as that occupied by the angle-of-attack probe. For example: if the angle-of-attack probe is located under the wing, then the normalization must be based on data directly from the angle-of-attack probe or at a location under the wing at a similar local chordwise and spanwise position.

Under these *steady* conditions the angle of attack simply increases or decreases along the calibration curve to support the changing weight or load factor (effective weight). Furthermore, the calibration 'curve' does not have to be linear. It can, in fact, be polynomial, exponential, etc. provided that it is not multiple valued with angle of attack as the dependent variable and a suitable normalized pressure ratio as the independent variable.

Guidance on choosing calibration points.

The discussion above strongly suggests using EAS_{CC}, $\text{EAS}_{L/D\text{max}}$ and a slightly higher value than $EAS_{P_{R_{\text{min}}}}$ plus one or two points between these values for determining the calibration 'curve' with a suitable normalized pressure ratio as the independent variable and angle of attack as the dependent variable. Because $EAS_{P_{R_{\text{min}}}}$ defines the dividing point between the frontside and backside of the thrust horsepower required curve for a piston-propeller aircraft a slightly higher value is suggested to insure that the aircraft is on the frontside of the thrust horsepower available curve and hence exhibits speed stability. A value of $0.8\,\text{EAS}_{L/D\text{max}}$ is suggested rather than the value of $0.76\,\text{EAS}_{L/D\text{max}}$ as indicated in Eq(13). The value for $\text{EAS}_{L/D\text{max}}$ at gross weight is readily available in Section 3 of the FAA standard aircraft pilot operating handbook (POH) as the best glide speed or speed for engine failure in flight. The POH best glide speed should be corrected for weight using Eq(15).

The calibration 'curve' should be determined within the angle of attack data acquistion system (DAS). If multiple calibration points, as suggested above, are used, then a regression analysis, coded into the DAS, should be used to determine the calibration 'curve'. The 'goodness' of the fit can be determined from the classical R-squared value. If the R-squared value is to low, the calibration runs should be repeated. A suitable visual notification should be provided, e.g., a green light for acceptable and/or an amber/red light for unacceptable calibration.

How should the calibration flight be conducted?

Each of the individual calibration data point runs should be conducted with fixed mixture, RPM, throttle and heading settings for a given data point. It is important to maintain a constant indicated airspeed. Slight continuous *steady* altitude increases or decreases are acceptable. If the aircraft is equipped with an autopilot, then use of the autopilot is suggested. A copilot to handle the data acqusition tasks, while the pilot concentrates on flying the aircraft, is recommended.

Key points.

- Absolute angle of attack *does not* vary with weight, density altitude, or load factor.
- Absolute angle of attack *does* vary with aircraft configuration, i.e., with parasite drag and Oswald aircraft efficiency factor, fe.
- The primary instrument used to develop data for a calibration 'curve', i.e., equivalent airspeed, EAS, *does* vary with weight and load factor but not density altitude. Hence, it should be corrected for effective weight, i.e., nW
- Suggested calibration points include EAS_{CC}, $\text{EAS}_{L/D\text{max}}$, $EAS_{P_{R_{\text{min}}}}$ and intermediate points.
- Pressure based angle-of-attack data acquisitions should use appropriatedly normalized pressure ratios.
- Normalization for pressure based angle-of-attack data acquisition systems should use a normalization pressure derived from a location either on the probe or located at a similar position on the aircraft.
- Multiple data points and regression analysis potentially yields more accurate calibrations 'curves'.

