A Performanced Based Angle of Attack Display

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The Problem

The current angle of attack displays basically warn you about the approach to stall with yellow and red indications and little else. Unfortunately, such displays contribute little or nothing to optimally flying the aircraft nor to indicating basic changes in other flight characteristics, e.g., between the regions of normal and reverse command (slow flight). However, with an accurate angle of attack system this is relatively easy to do. Recently, experiments have been undertaken to design an angle of attack display that is performance based in addition to warning of impending stall. An image of one such display is shown in Fig. 1

Accuracy Requirements

For a display such as shown to be useful an accuracy of between +/- 1/4 to +/- 1/2 a degree in angle of attack is required. If implemented digitally, the display requires appropriate damping to prevent the digits constantly flickering.

Mechanical angle of attack systems can provide the required accuracy. However, they are expensive and difficult to mount on a single engine aircraft without the requirement for a long wing mounted probe.

Differential pressure based angle of attack systems cannot provide this level of accuracy as currently typically implemented. The problems with differential pressure based probes are two fold:

lack of normalization to eliminate dynamic pressure effects;

the assumption of 2-point linear calibration with differential pressure.

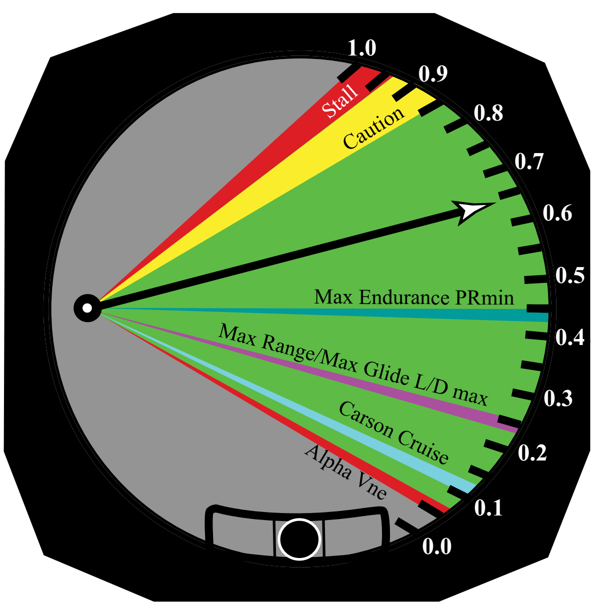

Figure 1. Circular performance based angle of attack display.

Fundamental Angle of Attack

The angle of attack for $L/D_{\max}$ is fundamental. The angle of attack for $L/D_{\max}$ can easily be shown to be independent of density altitude and weight. In fact, it depends only on aircraft design parameters. Specifically,

$$\alpha_{L/D_{\max}} = \frac{1}{a}\frac{b}{S}\sqrt{\pi f e}$$

where

- a is the lift curve slope;
- b is the wing span;
- f is the equivalent parasite drag area;
- e is the aircraft Oswald efficiency factor;
- π is $3.14159265\ldots$;
- S is the wing area.

f and e are dependent on aircraft configuration, e.g., gear up/down, flaps up/down, cowl flaps open, etc.

Two other angles of attack are of interest and are simple numerical multiples of the angle of attack for $L/D_{\max}$. Specifically, the angle of attack for minimum power required (maximum endurance) and the angle of attack for Carson Cruise.*

The angles are

$$\alpha_{P_{R_{\min}}} = \sqrt{3}\alpha_{L/D_{\max}} = 1.73\alpha_{L/D_{\max}}$$

and

$$\alpha_{\mathrm{CC}} = \frac{1}{\sqrt{3}}\alpha_{L/D_{\max}} = 0.58\alpha_{L/D_{\max}}$$

and are also independent of density altitude and weight.

Why Are These Angles Important?

In addition to the stall angle of attack they represent important performance conditions. The angle of attack for $L/D_{\max}$ represents both the best glide and the best range angle of attack.

The angle of attack for minimum power required represents the best endurance, i.e., the speed that provides the longest time aloft. In addition, it represents the boundary between the region of normal command and that for reverse command. In the region of reverse command in order to fly slower it is necessary to increase power available. This is counter intuitive. Furthermore, in the region of reverse command the aircraft no longer has speed stability.**

The angle of attack for Carson Cruise represents the minimum increase in fuel consumption for the maximum increase in speed (decrease in flight time). This is different and greater than the speed for maximum range. With fuel costs today, the Carson Cruise speed represents the minimum increase in fuel cost while still utilizing the decrease in time represented by air "travel".

A Better Way to Implement Differential Pressure Based Angle of Attack Systems

Dynamic pressure effects can be eliminated by using the differential pressure from the pitot-static system. This is undesirable for a number of reasons, e.g., regulatory, and undesirable negative effects on the sensitive airspeed and altitude instrumentation.

The errors from 2-point calibration, represented by the shaded area in Fig. 2, can be reduced by using a multi-point calibration technique as illustrated in Fig. 3 using a four-point

*www.nar-associates.com/technical-flying/efficiency/efficiency_wide_screen.pdf or the references therein.

**http://www.nar-associates.com/technical-flying/ speed_stability/speed_stability_wide_screen.pdf.

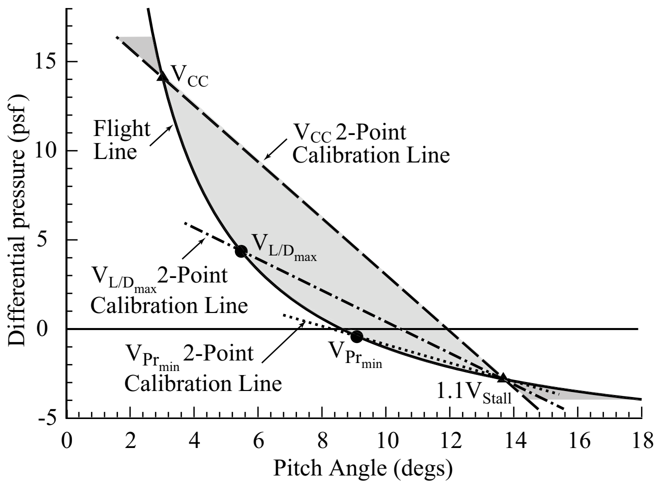

Figure 2. Flight line and the effect of 2-point linear calibration.

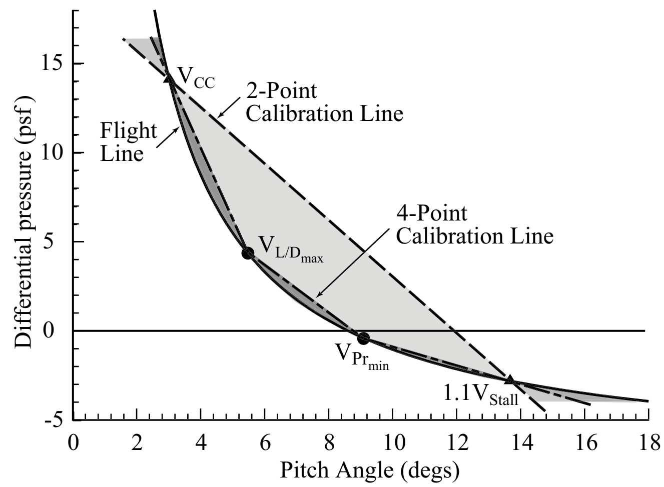

Figure 3. Comparison of 2-point and 4-point linear calibration.

calibration. Here, the errors are represented by the darker shaded area within the lighter shaded area. The four calibration points chosen, i.e., the absolute angles of attack for maximum lift to drag ratio, Carson cruise and minimum power required are independent of weight and density altitude.

An alternate technique for eliminating the dynamic pressure effects on the differential pressures uses a self-contained system.* The essence of the alternate technique is shown in Fig. 4. Wind tunnel tests show that normalizing the differential pressure with the pressure on the probe inclined surface eliminates the dynamic pressure effect. This eliminates the requirement to tap into the pitot-static system. Thus, either a four-point calibration scheme as shown in Fig. 3 with linear interpolation or the data from the wind tunnel tests or alternatively the on-board system can fit a parabolic curve to the calibration data. In either case the on-board system can simply solve the resulting parabolic equation for the differential pressure as a function of angle of attack. Enough computing power can easily, and inexpensively, be included in the display

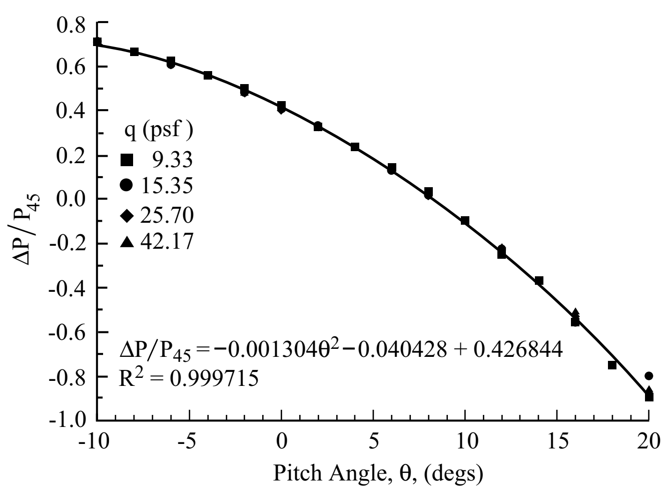

Figure 4. Alternate method of normalizing the differential pressure.

*Rogers, David F. "Wind Tunnel Investigation of a General Aviation Differential Pressure Angle of Attack Probe" AIAA Journal of Aircraft, Vol. 50, No. 5, pp. 1668-1672, September-October 2013. See also the article on the Technical Flying webpages—www.nar-associates.com/technical-flying/technical-flying.html#Angle.

system to simply solve the parabolic equation for the angle of attack in terms of the differential pressure. Alternatively, a simple table lookup, based on the wind tunnel data, can be used.

Display Considerations

With an accurate angle of attack system the display can take advantage of these additional performance and safety parameters. A number of presentations, e.g., a more or less traditional circular analog style presentation as shown in Fig. 1 and, for example, an analog style bargraph presentation. Here, an analog style implies continuous whether truly analog or digital.

Calibration Techniques

Four calibration points are needed to provide the required accuracy mentioned above. These are: $1.1V_{\text{stall}}$, $V_{P_{R_{\min}}}$, $V_{L/D\max}$ and V_{CC} all corrected for weight. The aircraft pilot operating handbook provides a calibrated airspeed for maximum lift to drag ratio or best glide speed. Typically this speed is for maximum gross weight. Assuming no airspeed instrument errors and small or negligible pitot-static system errors the indicated airspeed corrected for weight is

$$\text{IAS}_{L/D\max_{\text{current weight}}} = \sqrt{\frac{W_{\text{current weight}}}{W_{\text{gross weight}}}}\,\text{IAS}_{L/D\max_{\text{gross weight}}}$$

For example: if the gross weight is 3300 lbs, the current weight is 3000 lbs and the speed for best glide, $\text{IAS}_{L/D\max}$, at 3300 lbs is 121 mph, then

$$\text{IAS}_{L/D\max_{3000}} = \sqrt{\frac{3000}{3300}}\,121 = (0.9535)\,(121) = 115.4\,\text{mph}$$

Similarly, the velocities for $\text{IAS}_{P_{R_{\min}}}$ and IAS_{CC} are also simple numerical multiples of the $\text{IAS}_{L/D\max}$. Specifically,

$$\text{IAS}_{P_{R_{\min}}} = \frac{1}{\sqrt[4]{3}}\,\text{IAS}_{L/D\max} = 0.76\,\text{IAS}_{L/D\max} \quad\text{and}\quad \text{IAS}_{\text{CC}} = \sqrt[4]{3}\,\text{IAS}_{L/D\max} = 1.32\,\text{IAS}_{L/D\max}$$

Hence, they also vary with weight in the same manner as $\text{IAS}_{L/D\max}$. For the above example at 3000 lbs they are

$$\text{IAS}_{P_{R_{\min}}} = \frac{1}{\sqrt[4]{3}}\,\text{IAS}_{L/D\max} = 0.76\,\text{IAS}_{L/D\max} = (0.76)(115.4) = 87.7\,\text{mph}$$

and

$$\text{IAS}_{\text{CC}} = \sqrt[4]{3}\,\text{IAS}_{L/D\max} = 1.32\,\text{IAS}_{L/D\max} = (1.32)(115.4) = 152.3\,\text{mph}$$

The stall velocity also requires adjustment for weight. The stall velocity also varies as the square root of the weight. Specifically,

$$\text{IAS}_{\text{stall current weight}} = \sqrt{\frac{W_{\text{current weight}}}{W_{\text{gross weight}}}}\,\text{IAS}_{\text{stall gross weight}}$$

For the above example at 3000 lbs the adjusted stall speed in the clean configureation is

$$\text{IAS}_{L/D\max_{3000}} = \sqrt{\frac{3000}{3300}}\,72 = (0.9535)\,(72) = 68.6\,\text{mph}$$

Thus, $1.1\,\text{IAS}_{\text{stall}}$ is 75.5 mph.

If the angle of attack system is used with additional configurations, then the speed for $V_{L/D\max}$ and the stall speed in those configurations needs to be determined. Additional in-flight calibration for these additional configurations is required.

For example, for the power approach configuration, i.e., gear down, flaps 20°, cowl flaps open, from flight test, the product of fe for the example aircraft is 4.39. Hence, using the equations for indicated airspeed above at a weight of 3000 lbs the speed for maximum lift to drag ratio is 97 mph, the speed for minimum power required is 74 mph and the Carson cruise speed is 128 mph. From flight test, the stall speed at 3000 lbs is 61 mph. Hence, 110% of the stall speed is 67 mph. The angle of attack system then reqiures an additional in-flight calibration in that configuration at those four speeds. The second configuration calibration could be implemented either as a pilot activated manual switch or via a separate sensor activated by flap deflection and/or gear extension.

Summary

Current angle of attack displays lack any significant performance information other than stall awareness. The design of a performance based angle of attack display is proposed. A self-contained normalizaion scheme is presented which collapses the differential pressure to a single curve for all dynamics pressures. Errors in the displayed angle of attack are reduced by using either a 4-point calibration scheme based on 110% of stall speed, the speed for maximum lift to drag ratio, the speed for minimum power required and the Carson cruise speed with linear interpolation or direct solution of the parabolic equation for differential pressure versus angle of attack based on the self-contained normalization scheme.