Elevator Balance

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Have you ever wondered why Beech chose to use magnesium for the skins and ribs for the ruddervators on the V-tail and for the rudder and elevator on the straight tail Bonanza? Weight you say, and that's not wrong. But, why does the weight make a difference? Why does the Bonanza have such a nice feel to the controls? Why not use aluminum; other manufacturers did and do. After all, magnesium does exhibit corrosion problems, especially if a mixture of aluminum and magnesium are used; and it is more expensive than aluminum. The answer lies in feedback of control forces into the yoke for a manual unpowered reversible control system as used in the Bonanza. The explanation lies in the equations of motion of the controls during aircraft maneuvers.

The equations of motion of an aircraft in flight are many and complex. In fact, depending on the particular case being studied there may be as many as (or more than) 15 different equations, all of which have to be solved together. No, we are not going to look at all these equations. However, let's look at the one for the motion of the elevator. It is fairly simple and leads us to the explanation of why magnesium was chosen instead of aluminum, and to the reason that the balance of the ruddervators and elevator on the Bonanza is so important.

Elevator equation of motion: The elevator equation of motion is

$$F_e \times 1.0 = I_e\ddot{\delta}_e + m_e e_e a_{cg_z} + P_{ex}(PR - \dot{Q}) - H_e$$

This looks complicated. Let's take it apart term by term and make sense out of it. Referring to the schematic elevator in Figure 1 where each of these terms is illustrated will help the terms in the equation make physical sense. First, note that each of the terms in the equation represents a moment (or torque), i.e., a distance or length times a force, e.g., ft lbs.

The term $F_e \times 1.0$ is the elevator force. It is represented by the rectangular gray column with the arrow at the top in Figure 1. The 1.0 is a unit distance used to turn the force into a moment so that all the terms in the equation are consistent. F_e is the force that you exert when you pull or push on the yoke to move the elevator. It is also the force that the elevator feeds back into the yoke caused by aerodynamic and inertia forces that act on the elevator.

The first term to the right of the equal sign, $I_e\ddot{\delta}_e$, is the inertia moment, I_e, due to the angular

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acceleration of the elevator, $\ddot{\delta}_e$,[†] about the hinge line. Recall that inertia is the resistence to motion of an object and involves the mass of the elevator, m_e, of the object and the shape of the object.[††] The inertia moment resists inputs to the elevator through the yoke. Conversely, when the elevator is moved/accelerated by external forces, e.g., due to turbulence, the inertia moment feeds back forces, and hence motion, to the yoke.

The second term on the right side of the equal sign is the inertia moment, $m_e e_e a_{cg_z}$, due to linear acceleration in the z direction. Here, e_e, the eccentricity of the elevator (see Figure 1), is the distance that the center of mass of the elevator is behind the hinge line. The last symbol in the term, a_{cg_z}, is the acceleration of the aircraft center of gravity, cg, in the z direction. Notice from Figure 1 that the z-axis points out the belly of the aircraft and moves with the aircraft, i.e., even if the aircraft is rolled or pitched z still points out the belly. This is the term that nastily shakes the yoke in moderate or severe turbulence.

You can demonstrate this effect with a used hardback book. After opening the front cover of the book, clasp the book by the long front edge of the remaining pages with your thumb on top. Hold the book level in front of you and level with your shoulder with your arm slightly bent at the elbow. The open front cover should now be on the side away from your hand and hang slightly down toward the floor. Now abruptly move (accelerate) the book *straight* down toward the floor keeping your wrist rigid. What happened to the book's front cover? It flipped closed didn't it? No this is *not* caused by aerodynamics. It happens way to fast for that. It is an inertia effect. You can also simply drop the book. The effect is the same, but that makes a loud noise.

The third term, $PR - \dot{Q}$, is the inertial moment due to the combined angular rolling velocity, P, and angular yawing velocity, R, i.e., PR, of the aircraft and the pitch acceleration, $\dot{Q}$, on a little element of mass, dm, of the elevator (see Figure 1). Here, Q is the angular pitch velocity and hence $\dot{Q}$ is the pitching acceleration. P_{ex} is the combined product of inertia of all the little elements of mass, dm, of the elevator. Simplistically, while the inertia, I_e, is the resistance to motion (linear or angular) in one direction, the product of inertia, P_{ex}, can be thought of as the resistance to motion in two directions.

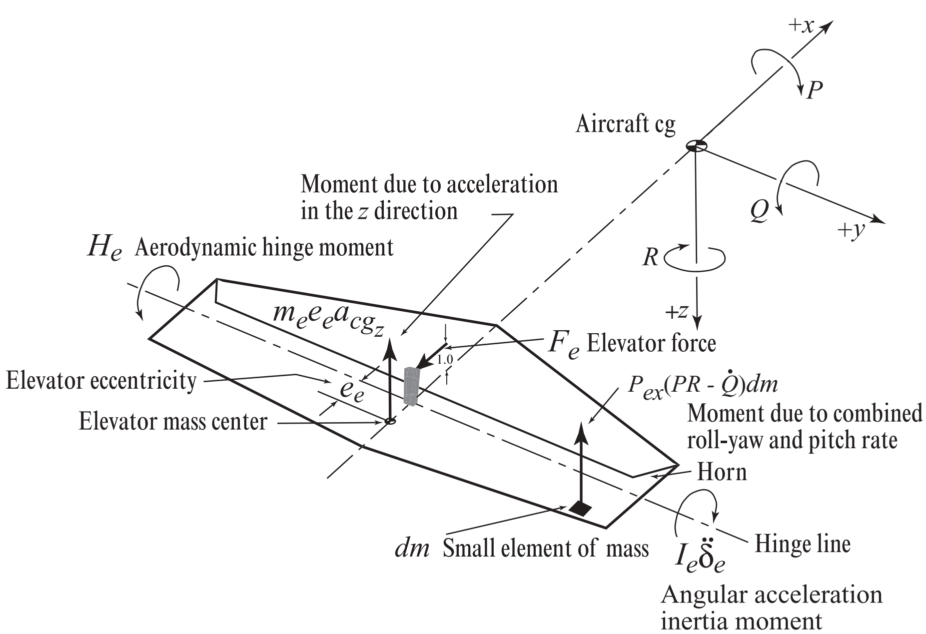

Figure 1. Elevator schematic

You can also demonstrate this effect with the hardback book. Again hold the book level in front of you, level with your shoulder with the front cover open. Now, with your arm stiff, rotate about your waist to the left — that's the yaw component. After the yaw rotation is established continue the yaw rotation and

[†] A dot over a symbol indicates change with time in the quantity represented by the symbol. Here one dot over the elevator angle, represented by δ_e, would indicate the rate at which the elevator angle was changing with time, i.e., the rotational velocity of the elevator. Two dots indicate the rate at which the rotational velocity of the elevator is changing, i.e., the rotational acceleration.

[††] The mass, m, of an object is the weight divided by the acceleration of gravity. Mass is a more fundamental quantity than weight, which varies with the gravitational field. For example, on the moon the weight of an object is approximately 1/6 th of the object's weight on earth. The mass is the same.

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rapidly roll your stiff arm counterclockwise — that's the roll component. The result is a combined yaw-roll, i.e., PR. What happened to the book cover? Again, it flipped closed, didn't it? Incidentally, notice that the cover flipped closed *against* any aerodynamic effect. This is a so called yaw–roll inertia cross-coupling. In a rolling yawing motion or a pitch acceleration, these inertia effects add to the force required to move or hold the elevator in position.

The last term, H_e, is the aerodynamic hinge moment. As the elevator is deflected the air flow around the elevator changes and produces not only a lift force which results in pitching of the aircraft but also, because the center of the lift force does not act at the hinge line, a moment about that hinge line. It is mostly this aerodynamic moment that you feel as a force at the yoke.

Mimimizing the inertia effects – dynamic balance: Now that we understand the source of the basically undesirable inertia forces and moments the question becomes how does a designer minimize them? The process of reducing the inertia effects is referred to as dynamically balancing the control.

We already pointed out that the moment of inertia of the elevator, I_e, depends on the mass of the elevator, m_e. The mass of magnesium is approximately 65% that of aluminum. By using magnesium rather than aluminum the mass of the elevator is reduced, the moment of inertia is reduced and hence the inertia moment due to angular acceleration, $I_e\ddot{\delta}_e$, along with the feedback into the yoke is reduced.

Considering the inertia moment due to linear acceleration in the z direction, $m_e e_e a_{cg_z}$, notice that if either or both the elevator mass and the elevator eccentricity are reduced, then the inertia moment is reduced and the feedback to the yoke is reduced. Hence, a designer attempts to reduce the eccentricity, e_e, as much as possible by using both appropriate layout of the physical elevator and weights to statically balance the control. Practically, the eccentricity cannot be made exactly zero and, on a manual reversible control system as on the Bonanza, having the center of mass of the elevator ahead of the hinge line is not desirable. Notice also that reducing the mass of the elevator reduces the inertia moment. By using magnesium for the skins and ribs, the designer gets an immediate reduction of 35% in the inertia moment. Furthermore, it is somewhat easier to reduce the eccentricity when statically balancing the control.

Eliminating the inertia moment effects due to combined rolling and yawing and pitch acceleration, $P_{ex}(PR - \dot{Q})$, is more difficult. Fundamentally, rolling yawing motions and pitch acceleration cannot be eliminated. Hence, the term $(PR - \dot{Q})$ cannot be made zero. Furthermore, the product of inertia P_{ex} involves both the mass of the elevator and the position of that little element of mass dm in Figure 1 in both x and y. It is relatively easy to show (you don't want me to do it) that for an elevator hinge line parallel to the y-axis (see Figure 1) P_{ex} cannot be made zero. Hence, for most light general aviation aircraft, including the Bonanza, the inertia moment due to combined rolling and yawing and pitch acceleration cannot be eliminated. However, by reducing the mass of the elevator by using magnesium instead of aluminum for the skins and ribs it is significantly reduced.

As an aside, if the elevator hinge line is swept, then it is possible to reduce the product of inertia, P_{ex}, and hence the inertia effect due to combined rolling-yawing and pitch acceleration.

These are the considerations that led the Beech designers to use magnesium for the elevator/ruddervators on the Bonanza. Similar equations of motion govern the ailerons as well as the rudder on the straight tail Bonanzas. It is no surprise that on the early Bonanzas magnesium was used for the ailerons. It is also used for the rudder on the straight tail Bonanza.

Aerodynamic balance: Aerodynamic balance is used to decrease the force on the yoke generated by the elevator. This is typically accomplished using an elevator horn. The horn is a portion of the elevator that projects forward of the hinge line (see Figure 1). When the elevator is deflected downward the horn is deflected upward. The aerodyanmic force on the upward deflected horn reduces the elevator hinge moment which results in a lower force being transmitted to the yoke. Similarly, when the elevator is deflected upward the horn is deflected downward and again the resulting aerodynamic force reduces the force transmitted to the yoke. Care must be taken in designing the shape and size of the horn. If the horn is improperly designed, the horn can generated too much aerodynamic force which results in the elevator feel becoming too 'light' or even slamming to the stops — neither is good!

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