Horseshoe Heading Technique

by

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Introduction

Using the Global Positioning System (GPS) to determine position error has many attractions. However, GPS determines ground speed and ground track resulting in significant errors if not used carefully [1,2].

Perhaps the first to suggest an accurate technique for determining position correction errors using GPS was David Fox writing in *Kitplanes*[2]. Fox's method required flying three perpendicular GPS tracks at the same indicated airspeed.

The National Test Pilot School[4], Rogers[1] and Smith[2] suggest flying directly into and away from the wind and averaging the resulting GPS ground speed. The difficulty in this technique is in determining the wind direction. Variation in wind direction and velocity contribute to sometimes unacceptable errors.

The United States Air Force Test Pilot School[5] suggests flying perpendicular to the wind in both directions along a GPS track, thus canceling the effect of the wind. Although this method reduces the errors due to variable wind direction and speed, the wind direction must still be determined.

Flight test personnel at the Federal Aviation Administration (FAA) suggest that after entering a GPS waypoint several thousand miles away the aircraft be flown directly towards and away from the waypoint. The difference between two fixed distances along the track and along the reverse direction is then timed with a stopwatch. With a large distance to the waypoint the aircraft follows nearly parallel tracks inbound and outbound between GPS distance rings, thus minimizing the error due to the wind. A variation of this technique (with smaller distances) was originally used at the United States Naval Academy.

Craig Cox at Rea Computing extended Fox's technique to use constant headings instead of constant tracks. This method, which has been called the horseshoe heading method, has significant advantages. For example, sequential constant headings at ninety degree intervals are easier to fly than constant tracks, headings can be in any direction as long as they are at ninety degree intervals, the method can be used at any reasonable altitude and the direction and magnitude of the wind does not need to be predetermined. The method was first published in June 1997 as a Java Applet on the REA website (*http://www.reacomp.com/true_airspeed*). Subsequently, at the request of users, the derivation of the equations was included on the website in February 2001. Although the derivation resulted in a quadratic equation in the square of the true airspeed, the Java Applet uses an iterative method to solve the equation. The derivation below, although independently obtained, closely follows that presented on the Rea Computing website. It is presented here for completeness. It is the technique currently used in the flight test course.

Doug Gray[6] extended Fox's basic method and the horseshoe heading method to arbitrary GPS tracks. For accuracy, the GPS tracks should be approximately ninety degrees apart. Although the aircraft heading, altitude and indicated airspeed are held constant, only the three GPS ground speeds and ground tracks are required to obtain a solution for the TAS.

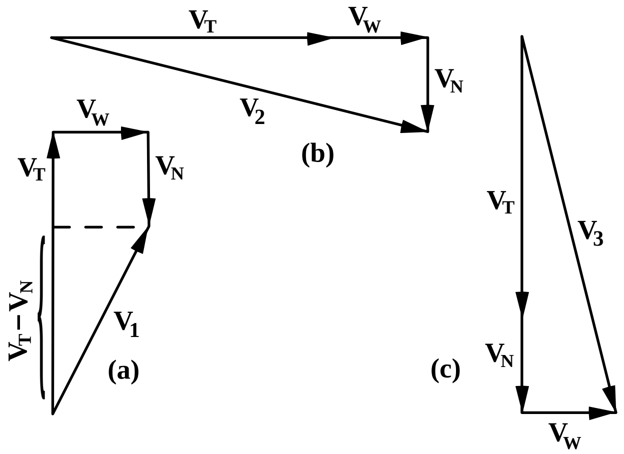

Figure 1. Horseshoe heading technique.

Horseshoe Heading Method

From Figure 1 parts (a), (b) and (c), respectively, the three equations in the three unknowns V_T, V_N and V_W are

$$(V_T - V_N)^2 + V_W^2 = V_1^2 \tag{1}$$

$$(V_T + V_W)^2 + V_N^2 = V_2^2 \tag{2}$$

$$(V_T + V_N)^2 + V_W^2 = V_3^2 \tag{3}$$

where V_T, V_N, and V_W are the true airspeed and northernly and westerly components of the wind, respectively. V_1, V_2, V_3 are the three measured GPS ground speeds for the three successive headings. Expanding and adding Eqs. 1 and 3 yields

$$2V_T^2 + 2V_N^2 + 2V_w^2 = V_1^2 + V_3^2 = P \tag{4}$$

Similarly, expanding and subtracting Eq. 3 from Eq. 2 yields

$$-2V_T V_N + 2V_T V_W = V_2^2 - V_3^2 = Q \tag{5}$$

Finally, subtracting Eq. 3 from Eq. 1 yields

$$-4V_T V_N = V_1^2 - V_3^2 = R \tag{6}$$

V_1, V_2 and V_3 are the known GPS ground speeds. Hence, P, Q and R are known; and Eqs. 4, 5 and 6 represent three equations in the three unknowns, V_W, V_N and V_T. Solving Eq. 6

$$V_T V_N = -\frac{R}{4} \tag{7}$$

Substituting Eq. 7 into Eq. 5 yields

$$2V_T V_W = Q - \frac{R}{2}$$

or

$$V_W = \frac{2Q - R}{4V_T} \tag{8}$$

From Eq. 7

$$V_N = \frac{-R}{4V_T} \tag{9}$$

Provided the true airspeed is available, the wind direction and magnitude are known from Eqs. 8 and 9. The true airspeed is obtained by substituting Eqs. 8 and 9 into Eq. 4 and expanding to yield

$$V_T^4 - \frac{P}{2}V_T^2 + \frac{2Q^2 - 2RQ + R^2}{8} = 0 \tag{10}$$

Equation 10 is a quadratic equation in V_T^2. An analytical solution is of course immediately available. Specifically

$$V_T = \sqrt{\frac{-b \pm \sqrt{b^2 - 4c}}{2}} \tag{11}$$

where $b = -P/2$ and $c = (2Q^2 - 2RQ + R^2)/8$.

Because a handheld aviation GPS typically displays the ground speed to only three significant figures, an analysis of the effect of GPS ground speed round-off error (GPS ground speed ± 1 kt) on the value calculated from Eq. 11 was examined for the case where the wind velocity was one half of the true airspeed and the wind direction was directly abeam the second leg. For general aviation aircraft true airspeeds – 75 to 200 kts – the maximum error in the calculated true airspeed is approximately one knot. Additional credibility for the horseshoe heading method is provided by inflight measurement by Lewis[4] at the National Test Pilot School using a Merlin III aircraft. Lewis, using a trailing cone for comparison, found excellent agreement with the horseshoe heading GPS method in determining position error corrections.

References

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