Estimating The Turnback Altitude

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Abstract

Turnback after engine failure is an edge of the operating envelope maneuver. A properly executed maneuver might save a pilot's life; improperly executed it likely ends it. Turnback after engine failure is a three-dimensional multiple variable performance problem with multiple geometric constraints. The turnback maneuver is cast as alternate asymptotic vertical plane boundary value problems. Both failure altitude and runway length are considered as unknown boundary conditions. The turnback maneuver is categorized on whether the failure altitude distance is more or less than four turn radii from the runway departure end. The performance elements of the maneuver are estimated from data in the pilot operating handbook (POH). Three aircraft, an E33A Bonanza, a Cessna 172M and an Aeronca 7AC, are used as illustrative examples. The ratio of the climb to glide flight path angle emerges as the predictor of success, or failure. If the ratio is more than one, a properly flown maneuver is generally successful. If the ratio is less than one, a successful maneuver is generally limited to a small range of failure altitudes and runway lengths or is unsuccessful. This ratio, easily determined from POH information, strongly predicts success or failure. Pilots should know which option physics favors.

Introduction

The turnback after engine failure discussion has been on going for literally decades, if not longer. Up until recently the 'official' wisdom has been to not attempt a turnback to the departure runway. Recently, the FAA published guidance for flight instructors (AC 61-83J Sec A.11.4) with respect to the "demonstration and teach(*ing of*) trainees when and how to make a safe 180-degree turnback to the field after an engine failure".*

However, The FAA provided little guidance in the advisory circular on how to accomplish this maneuver. Fortunately, scientific and engineering guidance has been available in the technical literature since at least 1982 [1-3]. Both the 1982 Jett simulator study and the 1995 Rogers papers clearly show that the optimum conditions for the turn itself are a bank angle of 45° at a speed as close to stall (maximum $C_{L\max}$) as possible. The Rogers 2012 paper [3] illustrates the penalties associated with *not* executing the maneuver using the optimal conditions.

In fact, in 1974 Schiff [4] and two professional pilot colleagues conducted a series of informal flight tests with five different light general aviation aircraft to determine the effect of bank angle on the loss of altitude during a 180° turnback turn. Specifically Piper PA-18A-150 Super Cub, Piper PA-28-140 Cherokee 140, Cessna 150 Aerobat, Cessna 172L Skyhawk and Cessna 185 (with cargo pod) were used. Four bank angles, 30°, 45°, 60° and 75° were tested. The tests results showed that a 45° bank angle was optimal. In those flight tests no mention is made of the speed used in the turns.

The turnback maneuver is a three-dimensional multiple variable performance problem with significant geometric constraints. There are a number of 'free' parameters/variables in the problem, e.g., climb flight path angle, glide flight path angle, failure altitude, runway length, bank angle, turnback turn angle and the lift coefficient/angle of attack/speed in the turns. The lift coefficient/angle of attack/speed are equivalent ways of specifying the manner of executing the turn. There is also the effect of wind. Rogers [2-3] addressed the effects of flight path climb angle, speed and wind. Rogers showed that it is more advantageous to climb at the speed for maximum climb angle than at the speed for maximum rate of climb. Climbing at the speed for maximum climb angle keeps the aircraft closer to the runway [2]. It is also more advantageous to execute the turnback turn close to $C_{L\max}$ and in a 45° bank, as shown by Eq. (1) below and the Rogers 2012 paper [3].

The Mathematical Model

Fundamentally, the turnback maneuver is a three-dimensional two point asymptotic boundary value problem.

A simplified model was previously used [1], consisting of four elements, takeoff to 50 ft, climb at a specified velocity and rate of climb, a circular turn through a specific heading angle at a specified bank angle and lift coefficient/angle of attack/speed and a maximum performance glide to the end of the departure runway. An enhanced model retains the first three elements of the simplfied model. In the enhanced model the glide is to the tangent to a circular realignment turn which is also tangent to the runway centerline. The enhanced model accounts for the altitude loss in the circular realignment turn to the runway. It also allows for an altitude 'cushion' over the runway centerline.

Using the enhanced model, the three-dimensional problem is reformulated as multiple coupled two point asymptotic boundary value problems. The enhanced model assumes small descent angles and fixed geometric flight paths over the two-dimensional ground plane. Thus, the enhanced model reduces the asymptotic boundary value problems of interest to those in the vertical plane. Two alternate asymptotic boundary value problems are of interest:

> Given a fixed runway length, what failure altitudes, if any, result in an acceptible altitude over the centerline of the runway? Here, the failure altitude is the unknown boundary condition.
>
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The Basis For The 45° Bank At $C_{L\max}$

What is the basis for using the optimal 45° bank angle and a speed as near to $C_{L\max}$ as possible during the turn?

*The entire Sec A.11.4 of AC 61-83J is included in Appendix A.

The optimal conditions for minimum loss of altitude (h) in a steady state gliding turn to a new heading (Ψ) come from the following equation [1-3]

$$\frac{dh}{d\Psi} = \frac{C_D}{C_L^2} \ \frac{4W}{\rho S g} \ \frac{1}{\sin 2\phi} \tag{1}$$

The equation assumes a parabolic drag polar.

This equation tells us what we need to know about the aircraft and the flight conditions in order to estimate the altitude loss in the turn. Specifically, the equation tells us that we need to know:

C_D, the drag coefficient;
C_L, the lift coefficient;
W, the weight;
S, the wing area;
g, the acceleration of gravity;
ρ, the air density;
ϕ, the bank angle.

Now, let's look more closely at the above list of parameters. In a steady descending turn at constant velocity and bank angle all those parameters (values) are constant. Note: the density change for a few hundred feet is small enough to ignore.

Furthermore, looking even more closely, we notice that dh and $d\Psi$ indicate that this is a differential equation. Thus, in order to determine the altitude loss for a given heading change, the equation needs to be *integrated*. Oops. But, not to worry.

Because all those variables are constant they can be replaced by a single number that we will call Constant and can be moved outside of the integral sign. Thus, we have

$$\int_{h_1}^{h_2} dh = \left[\frac{C_D}{C_L^2} \ \frac{4W}{\rho S g} \ \frac{1}{\sin 2\phi}\right] \int_{\Psi_1}^{\Psi_2} d\Psi = \text{Constant} \int_{\Psi_1}^{\Psi_2} d\Psi \tag{3}$$

where Constant is just a number. After integration, the result is just

$$h_2 - h_1 = \text{Constant}(\Psi_2 - \Psi_1) \text{ or } h_t = \text{Constant } \Delta\Psi \tag{4}$$

where h_t is the altitude loss in the turn and $\Delta\Psi$ is the change in heading during the turn. Thus, determining the loss of altitude in the turnback turn reduces to evaluating Constant for a specific heading change Ψ.

Extracting Information From The Pilot Operating Handbook

ESTIMATING THE LIFT COEFFICIENT

The gross weight, W, and wing area, S, are readily available in the POH (Pilot Operating Handbook). The air density, ρ (0.002377 slugs/ft^3 at sea level) is easily calculated [4]. The acceleration of gravity, g, is 32.174 ft/sec^2. The bank angle, ϕ, is optimally 45°.* What remains are the lift coefficient, C_L, and the drag coefficient, C_D.

The lift coefficient is easily determined from the clean (flaps and gear up) stall speed in the POH. Specifically,

$$C_L = \frac{1}{{}^1\!/\!_2\rho V^2} \frac{W}{S} = \frac{1}{{}^1\!/\!_2\sigma\rho_{\text{SSL}}V^2} \frac{W}{S} = \frac{1}{{}^1\!/\!_2\rho_{\text{SSL}} EAS^2} \frac{W}{S} \tag{5}$$

where V is the true airspeed, EAS is the equivalent airspeed and ρ_{SSL} is the standard sea level air density.

To illustrate, let's look at three typical light general aviation single engine aircraft: a four seat high performance low wing retractable, e.g., a 1969 E33A Bonanza; a four seat high wing fixed gear aircraft, e.g., a 1974 Cessna 172M and

*Be very careful to use consistent units, e.g., speeds in ft/sec, weight in lbs, areas in ft^2.

a high wing tandem two seat fixed gear aircraft, e.g., a 1946 Aeronca 7AC. The C172M is typically equipped with wheel pants while the 7AC is not.**

1969 Beechcraft E33A Bonanza

From the POH the clean stall speed for an E33A Bonanza is 72 mph or 105.6 ft/sec (72(88/60)) = 105.6). Hence, with a wing area, S, of 181 ft^2 and a gross weight, W, of 3300 lbs the lift coefficient is

$$C_L = \frac{1}{1/2(0.002377)(105.6)^2}\frac{3300}{181} = 1.376 \tag{6}$$

1974 Cessna 172M

Similarly, for a 1974 Cessna 172M, the POH gives the clean stall speed as 57 mph or 83.6 ft/sec with a gross weight of 2300 lbs. The Cessna wing area is 174 ft^2. Thus, the lift coefficient is

$$C_L = \frac{1}{1/2(0.002377)(83.6)^2}\frac{2300}{174} = 1.591 \tag{7}$$

Aeronca 7AC

Finally the POH for the Aeronca 7AC gives the stall speed as 38 mph or 55.73 ft/sec. The POH for the Aeronca 7AC also gives a wing area, S, of 170.22 ft^2 and a gross weight, W, of 1220 lbs.† Hence,

$$C_L = \frac{1}{1/2(0.002377)(55.73)^2}\frac{1220}{170.22} = 1.942 \tag{8}$$

With the maximum lift coefficent in hand we now turn to the drag coefficient.

ESTIMATING THE DRAG COEFFICIENT

Estimating the drag coefficient, C_D from information in the POH is a bit more difficult. The POH gives the speed for maximum glide, typically in the emergency section. The $\mathrm{EAS}_{L/D\mathrm{max}}$ given in the emergency section is with the propeller in the most optimal position and windmilling. The speed for maximum glide is the speed for maximum lift to drag ratio. The speed for maximum glide ratio is given by

$$V_{L/D\mathrm{max}} = \left(\frac{2}{\rho}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}\right)^{1/2} \quad \text{or} \quad \mathrm{EAS}_{L/D\mathrm{max}} = \left(\frac{2}{\rho_{\mathrm{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}\right)^{1/2} \tag{9}$$

and

$$(\mathrm{EAS}_{L/D\mathrm{max}})^2 = \frac{2}{\rho_{\mathrm{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}} \tag{10}$$

Notice that $\mathrm{EAS}_{L/D\mathrm{max}}$ is dependent only on aircraft design parameters, W and b, and aircraft configuration parameters, f and e. Notice also that it is not dependent on the local air density but only on ρ_{SSL}, the air density at sea level on a standard day.

Solving Eq. (10) for f, the equivalent parasite drag area, results in

$$f = \frac{1}{\pi e}\left[\frac{2}{\rho_{\mathrm{SSL}}}\;\frac{W}{b}\;\frac{1}{(\mathrm{EAS}_{L/D\mathrm{max}})^2}\right]^2 \tag{11}$$

We now see that the wing span, b, is required along with e, the so called Oswald aircraft efficiency parameter. The wing span, b, is available from the POH; but how do we obtain the value (number) for e? Here is where estimation comes to the forefront. The Oswald aircraft efficiency [5], e, typically has a value between 0.65 and 0.75. Occasionally it can be as low as 0.5 for very 'dirty' aircraft or as high as 0.8 for very 'clean' aircraft. Let's assume a value of 0.7 and test it against a known flight test value for the equivalent parasite drag area, f, for a known aircraft [6].

**The parameters for these three aircraft are shown in Table 1 in Appendix A.

†I've seen gross weights from 1200 to 1320 lbs.

Beechcraft Bonanza E33A

For an E33A the wing span, b, is 33.5 ft and the speed for $\text{EAS}_{L/D\text{max}}$ is 122 mph or 178.9 ft/sec. With these numbers and estimating e as 0.7, Eq. (11) becomes*

$$f = \frac{1}{0.7\pi}\left[\frac{2}{0.002377}\;\frac{3300}{33.5}\;\frac{1}{(178.9)^2}\right]^2 = 3.047\,\text{ft}^2 \tag{12}$$

From flight test results [6] for an E33A, the equivalent parasite drag area, f, is 3.125 ft^2.† Hence, the estimated value is within approximately 2.5% of the flight test result.**

With these results the E33A drag coefficient at maximum lift coefficient is

$$C_D = C_{D_o} + \frac{C_L^2}{\pi ARe} = f/S + \frac{C_L^2}{\pi(b^2/S)e} = 3.04/181 + \frac{(1.376)^2}{\pi((33.5)^2/181)0.7} = 0.0168 + 0.1389 = 0.1557 \tag{13}$$

Thus, Eq. (1) gives the estimated altitude loss in the typical 210° gliding turn [2] in a 45° bank at $C_{L\text{max}}$

$$\begin{aligned} h_t &= \left[\frac{C_D}{C_L^2}\;\frac{4W}{\rho Sg}\;\frac{1}{\sin 2\phi}\right]\Delta\Psi = \left[\frac{0.1557}{(1.376)^2}\;\frac{4(3300)}{0.002377(181)(32.174)}\;\frac{1}{\sin(90)}\right](210/57.296) \\ &= (0.08223)(953.4887)(1)(3.665) \\ &= 287.4\,\text{ft} \end{aligned} \tag{14}$$

The 57.296 $(180/\pi)$ is to convert from degrees to radians. It makes the units come out correctly.

Cessna 172M

Now let's turn our attention to the Cessna 172M. The Cessna 172M is a typical high wing four place aircraft. The 172M has a wing span, b, of 36 ft. From the emergency section of the POH $\text{EAS}_{L/D\text{max}}$ is 80 mph or 117.3 ft/sec. With these numbers and again using $e = 0.7$ Eq. (11) becomes

$$f = \frac{1}{0.7\pi}\left[\frac{2}{0.002377}\;\frac{2300}{36}\;\frac{1}{(117.3)^2}\right]^2 = 6.94\,\text{ft}^2 \tag{15}$$

From this result, the Cessna 172M drag coefficient at maximum lift coefficient is

$$\begin{aligned} C_D &= C_{D_o} + \frac{C_L^2}{\pi ARe} = \frac{f}{S} + \frac{C_L^2}{\pi(b^2/S)e} = \frac{6.94}{174} + \frac{(1.591)^2}{\pi((34)^2/174)0.7} = \frac{6.94}{174} + \frac{(1.591)^2}{\pi((34)^2/174)0.7} \\ &= 0.040 + 0.154 \\ &= 0.1945 \end{aligned} \tag{16}$$

Thus, for a Cessna 172M the estimated altitude loss in the typical 210° turn in a 45° bank at $C_{L\text{max}}$, on a stand day at sea level Eq. (1) yields

$$\begin{aligned} h_t &= \left[\frac{C_D}{C_L^2}\;\frac{4W}{\rho Sg}\;\frac{1}{\sin 2\phi}\right]\Delta\Psi = \left[\frac{0.1945}{(1.591)^2}\;\frac{4(2300)}{0.002377(174)(32.174)}\;\frac{1}{\sin(90)}\right](210/57.296) \\ &= (0.07679)(691.36)(1)(3.665) \\ &= 194.6\,\text{ft} \end{aligned} \tag{17}$$

Aeronca 7AC

*All calculations in the paper are at sea level on a standard day unless otherwise explicity stated.

†As is typical of POHs, the value in later POHs is slightly different, e.g., 121 mph or 177.5 ft/sec. Hence, $f = 3.09\,\text{ft}^2$. Thus, in this case the result is within approximately 1.1% of the flight test value.

** To match the flight test result using $f = 3.047\,\text{ft}^2$ the estimated e value decreases to 0.683, C_D increases to 0.1595 and h_t to 294.6 ft.

Finally let's consider the Aeronca 7AC. The 7AC is a typical high wing two place tandem seating training aircraft. The 7AC has a wing span, b, of 35 ft 1 3/4 in. From the POH, $\text{EAS}_{L/D\max}$ is 60 mph or 88 ft/sec. With these numbers and again using $e = 0.7$ Eq. (11) becomes

$$f = \frac{1}{0.7\pi}\left[\frac{2}{0.002377}\ \frac{1220}{35.1458}\ \frac{1}{(88)^2}\right]^2 = 6.45\,\text{ft}^2 \tag{18}$$

However, the 7AC typically is operated without wheel pants which increases the drag. Let's adjust the value of e downward to 0.6 to compensate for the additional drag because of the lack of wheel pants. The result is simply $6.45(0.7/0.6) = 7.55\,\text{ft}^2$.

Using an Oswald aircraft efficiency, $e = 0.6$, an equivalent parasite drag area of 7.75 ft^2 and Eq. (5), the Aeronca 7AC drag coefficient at maximum lift coefficient is

$$C_D = C_{D_o} + \frac{C_L^2}{\pi ARe} = f/S + \frac{C_L^2}{\pi(b^2/S)e} = 7.55/170.22 + \frac{(1.94)^2}{\pi((35.1458)^2/170.22)0.6} = 0.0443 + 0.2755 = 0.320 \tag{19}$$

Eq. (1) then yields an estimated altitude loss in the typical 210° turn in a 45° bank at $C_{L\max}$ for the 7AC of

$$\begin{aligned} h_t &= \left[\frac{C_D}{C_L^2}\ \frac{4W}{\rho Sg}\ \frac{1}{\sin 2\phi}\right]\Delta\Psi = \left[\frac{0.320}{(1.94)^2}\ \frac{4(1220)}{0.002377(170.22)(32.174)}\ \frac{1}{\sin(90)}\right](210/57.296) \\ &= (0.085)(374.87)(1)(3.665) \\ &= 116.6\,\text{ft} \end{aligned} \tag{20}$$

This altitude loss is considerably less than for either the E33A or the C172M. Notice however that C_D/C_L^2 is approximately the same for all three aircraft, i.e., 0.082, 0.077 and 0.085 for the E33A, C172M and 7AC respectively, with an average value of 0.081.

However, the wing loading, W/S, for the three aircraft is significantly different i.e., 18.2 lbs/ft^2 for the E33A, 13.2 lbs/ft^2 for the C172M and 7.75 lbs/ft^2 for the 7AC. In fact, altitude loss increases linearly with the wing loading as shown by Eq. (1).

The Rest of The Turnback Maneuver Estimate

At this point it is convenient to think about the turnback maneuver as six linked maneuvers:

takeoff and climb to 50 ft;
climb from 50 ft to a failure altitude;
a turn back toward the airport;
a glide, if any, toward the airport;
a turn to realign the aircraft flight path to a runway;
landing from a specified 'cushion' altitude.

An instantaneous transition is assumed between each of the elements.

To address the rest of the turnback maneuver, the simplified model used by Rogers [2] with the addition of a realignment turn is assumed. Specifically, the simplified model [2] used data from the manufacturer's pilot operating handbook (POH) for the subject aircraft to determine the initial take-off ground roll, rotation and lift-off velocities and the distance over a 50 foot obstacle. An instantaneous transition from the velocity at 50 feet to the specified climb out velocity was assumed. A steady climb at constant velocity from 50 feet to the failure altitude while maintaining runway heading was assumed. At engine failure, an instantaneous transition to a banked unpowered descending gliding turn at the assumed bank angle and the assumed velocity through an assumed heading change was used. Upon completion of the turn an instantaneous transition to the velocity for $L/D_{\max}$ was assumed. The current enhanced simplified model makes the same assumptions but changes the glide at $V_{L/D\max}$ to the tangent to a realignment turn. At that point, the enhanced simplified model assumes an instantaneous transition to a realignment turn at an assumed bank angle and velocity to align the aircraft with a runway. At the end of the unpowered glide

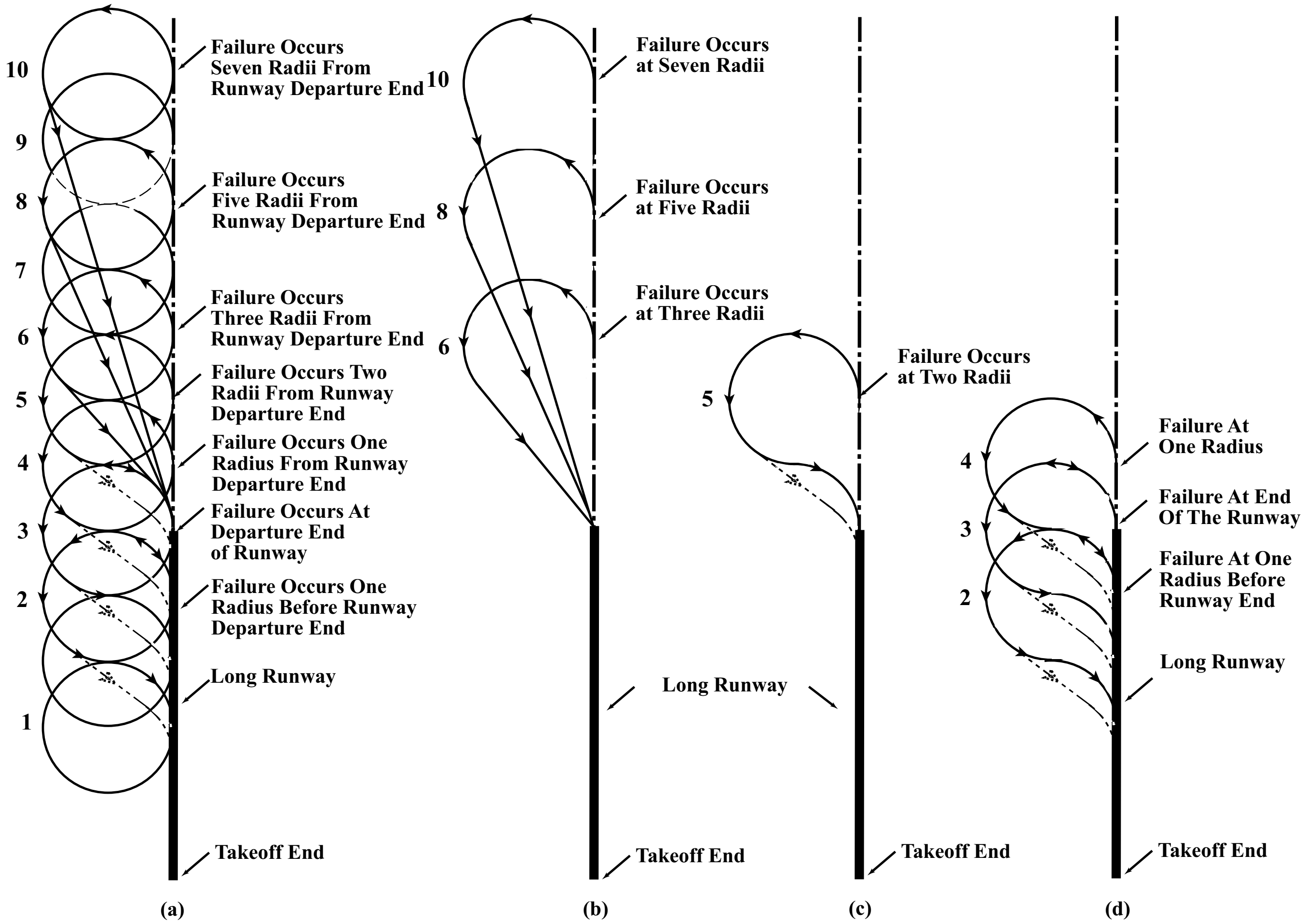

Figure 1. Possible turnback scenarios.

Rogers [2] did not consider any altitude cushion. Here, the enhanced simplified model assumes a minor altitude cushion at the end of the realigment turn to allow for pilot imperfect maneuver execution and landing effects.

CONCEPTUALIZING THE TURNBACK MANEUVER

The basic turnback maneuver is conceptualized in terms of the radius of the turnback turn as shown in Fig. 1a. Specifically, three cases are used: far from the departure end of the runway, near the departure end of the runway, and at or before the departure end of the runway. Figure 1a shows ten circles (dashed lines labelled 1-10) and nine turnback maneuvers (labelled 2-10) at various distances from the departure end of a 'long' runway. For illustrative purposes Fig. 1a shows both the turnback turn and the realignment turn with the same radius. Hence, both the turnback turn and the realigment turn are assumed to be flown at the same airspeed and bank angle. Figure 1a also shows the flight path after the turnback maneuver for six of the ten turnback circles. The flight path is represented by the solid lines with solid arrow heads. Unless otherwise stated, it is assumed that the pilot heads for the departure end of the runway upon completion of the turnback turn. Alternate flight paths are shown by chained dashed lines with dashed arrow heads when the pilot is assumed to head other than for the departure end of the runway.

In the first case, circles ten, eight and six in Figs. 1a and 1b represent turnback maneuvers with an unpowered glide segment, followed by a realignment turn to the departure end of the runway. This is the 'classic' turnback maneuver [1-3]. From Fig. 1b it is clear that as the failure altitude and the turnback turn approaches the departure end of the runway the realignment angle increases.

The second case is illustrated by maneuver five in Fig. 1a and 1c. For maneuver five, the failure altitude occurs at a distance of two radii of the turnback turn beyond the departure end of the runway. Circle five is the limit where the pilot has the option of heading for the departure end of the runway. Two flight paths are shown for this case:

> The first flight path is shown by the solid line and the solid arrow heads in Figs. 1a and 1c, illustrates the case where the turnback turn is executed through 270°. The 270° turn is immediately followed by reversing the 45° bank angle and executing the realignment turn through 90° to land on the departure end of the runway. Here, the entire altitude loss is due to the 360° of total turning.
>
> The second flight path is shown in Figs. 1a and 1c by the solid line and the solid arrow head followed by the chain dashed line and dashed arrow head. Here the turn is executed through 225°. The 225° turn is followed by a glide to a 45° realignment turn to land *near* the departure end of the runway. The total altitude loss results from 270° of turning (225° + 45°) plus the altitude lost in the glide between the turns. Because the altitude loss in an unbanked glide is typically less than in a banked gliding turn, the total altitude loss is likely less than in 360°s of turning. Considering that aircraft landing distances are typically less than aircraft takeoff distances, a successful landing may result depending on runway length.

In the third case, circles four, three and two in Figs. 1a and 1d, the failure altitude occurs before two radii from the departure end of the runway or earlier. Here the results are similar to those where the failure altitude occurs at two radii (circle five) from the departure end of the runway.

Again, two possible flight paths are shown. However, in this case, i.e., circles four, three and two, the landing occurs *before* the departure end of the runway. Hence, the remaining length of the runway available for landing becomes important.

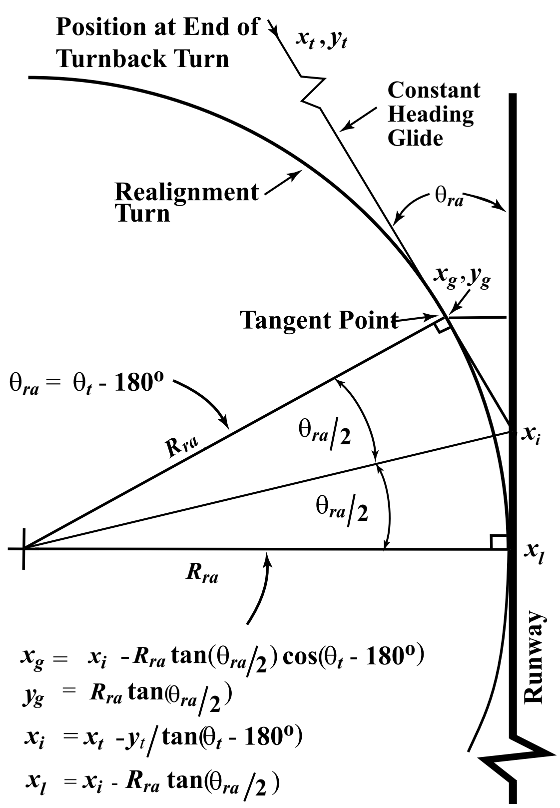

Figure 2. Direct heading geometry.

Given a sufficiently long runway, a fourth case, not shown explicitly in Fig. 1 occurs. Here, the pilot executes a 'classic' turnback turn through approximately 210°. At the end of the turnback turn the pilot maintains heading and executes a glide at $L/D_{\max}$ until nearly intersecting the runway. At that point a realignment turn at an appropriate bank angle and speed is executed. This is known as the Direct case. Here, as the geometry shown in Fig. 2 illustrates, the realignment turn angle is the turnback turn angle minus 180°. For example: if engine failure occurs at approximately four radii ($4R_t$) from the departure end of the runway, then for a turnback turn of 210° the flight path intersection angle with the runway centerline is 30°. The realignment turn angle is 30° as illustrated in Fig. 2. The total turning angle is thus 240°. For turnback turn angles less than approximately 210°, both excess glide distance and runway length are required. For turnback turn angles more than approximately 225°, excessive altitude loss in the turnback turn results.

The basic turnback geometry shown above in Fig. 1 may vary depending on the speed and bank angle the turns are flown as well as the speed of the glide, if any. Also, radii of the circles may not be equal if the turns are flown at different speeds and bank angles. However, the basic categorization is the same.

SOME EXAMPLES.

Using the three aircraft discussed above as examples is instructive. The altitude lost in the turnback turn and the realignment turn is given by Eq. (1) in the above discussion.

The turn back toward the airport occurs at a distance away from the runway takeoff end called the failure distance, x_f, and at some altitude called the failure altitude, h_f. In order to determine the failure distance some additional information from the POH is necessary. Specifically: $V_{R/C\max}$ or V_y, the speed for maximum rate of climb and $R/C_{\max}$, the maximum rate of climb at V_y are required.

1969 Beechcraft Bonanza E33A

From the POH for the E33A the velocity for maximum rate of climb, $V_{R/C\max}$, is 112.5 mph or 165 ft/sec while the maximum rate-of-climb, $R/C_{\max}$ is 1200 ft/min or 20 ft/sec at gross weight at sea level on a standard day. From Fig. (3) the climb flight path angle, γ_c, is

$$\gamma_c = \sin^{-1}\left(\frac{h_c}{d_c}\right) = \sin^{-1}\left(\frac{R/C_{\max}}{V_{R/C\max}}\right) = \sin^{-1}\left(\frac{20}{165}\right) = 6.96° \tag{21}$$

Referring to Fig. 3, the aircraft distance from the takeoff end of the runway from an altitude of 50 ft, x_{50}, to the failure distance from the takeoff end of the runway, x_f, while climbing along the flight path, d_c, is x_c. Hence,

$$x_c = d_c \cos\gamma_c \tag{22}$$

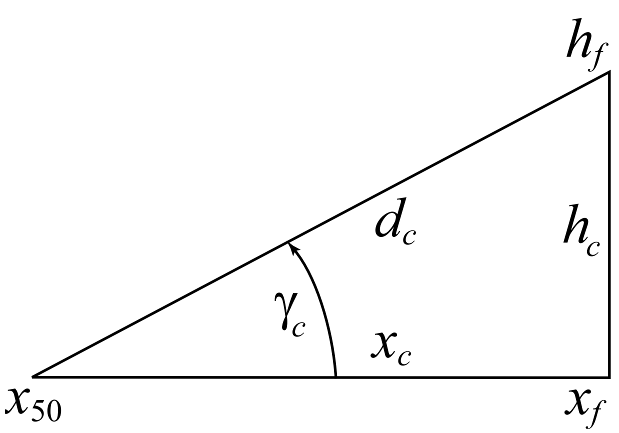

Figure 3. Climb diagram for an E33A Bonanza.

If we know the time, t_c, it takes to climb from 50 ft to the failure altitude, h_f, and the speed for maximum rate of climb, $V_{R/C\max}$, and the rate of climb, then the distance from 50 ft, $d_c \approx x_c$ is just

$$x_c = \frac{h_c}{R/C_{\max}} V_{R/C\max} = t_c\, V_{R/C\max} = \frac{685-50}{20} 165 = (31.75)(165) = 5239\,\text{ft} \tag{23}$$

In order to determine the distance down range from the start of the takeoff run we assume a turnback failure altitude, h_f. Using the simplified model, Rogers [2] assumed a failure altitude of 650 ft AGL (Above Ground Level). The simplified model did not allow for a realignment turn nor for any cushion. Here, using the enhanced model, a failure altitude of 685 ft is assumed. From the E33A POH we know the takeoff distance over a 50 ft obstacle is $x_{50} = 1750\,\text{ft}$. Thus, it takes 31.75 seconds to climb from 50 ft to 685 ft. The resulting total estimated ground distance from the beginning of the takeoff run is $x_f = 5239 + 1750 = 6989\,\text{ft}$, i.e., more than a nautical mile.

Rogers [2] indicates that climbing at V_x, the speed for maximum climb angle, is more advantageous. Climbing at the speed for maximum climb angle keeps the aircraft closer to the airport. Hence, after the turn, less glide distance is required to reach the runway. Unfortunately, the typical POH does not provide the rate of climb at V_x. Hence, the climb from 50 ft to the failure altitude is assumed to occur an the speed for maximum rate of climb, $V_{R/C\max}$ or V_y.

The Unpowered Glide To The End Of The Runway

E33A Bonanza

Figure 4 is a scaled drawing of the takeoff, climb, turnback turn and realignment turn for an E33A departing from a typical general aviation 3000 ft runway. Again, a standard no wind day at sea level is assumed. In addition, it is assumed that the pilot heads for the departure end of the typical 3000 ft runway. The diagram clearly illustrates that the turnback turn is about 210° and that the realignment turn is of the order of 10°. Clearly this corresponds to the 'classic' turnback case, Case 1, as discussed above.

These calculations lead us to the next piece of the problem, specifically the unpowered glide back to the runway.

Determining the glide distance to the end of the runway and the realignment turn angle requires some simple geometry as shown in Fig. 5. The radius of the unpowered gliding turn back to the runway end at $C_{L\max}$ in a 45° bank is

$$\begin{aligned} R_t &= \frac{V^2}{g\tan\phi} \quad \text{or in terms of } C_L \\ R_t &= \frac{1}{g\tan\phi}\,\frac{2}{\sigma\rho_{\text{SSL}}}\,\frac{W}{S}\,\frac{1}{C_L} = \frac{1}{(32.174)(1)}\,\frac{2}{(1)(0.002377)}\,\frac{3300}{181}\,\frac{1}{1.376} \\ &= 346.5\,\text{ft} \end{aligned} \tag{24}$$

where, for clarity, the radius of the turn, R_t, is rewritten in terms of the lift coefficient $C_L = (W/S)/(2\sigma\rho_{\text{SSL}}V^2)$.* Here, $\sigma = \rho/\rho_{\text{SSL}}$, is the atmospheric density ratio. For an E33A at maximum lift coefficient at sea level $C_{L\max} = 1.376$ (see Eq. (6)) and $\sigma = \rho/\rho_{\text{SSL}} = 1.0$.

From Fig. 5 the lateral offset from the extended runway centerline, y_t, for a 210° turnback is

$$y_t = R_t(1 + \cos(30^\circ)) = (346.6)(1 + 0.866) = 646.6\,\text{ft} \tag{25}$$

If the E33A climbs at V_y, the distance from the departure end of a 3000 ft runway is

$$x_t = x_c + x_{50} - x_r - R_t\sin(30^\circ) = 4950 + 1750 - 3000 - (346.5)(0.5) = 5238.8 + 1750 - 3000 - 173.3 = 3815.5\,\text{ft} \tag{26}$$

The realignment angle, θ_{ra}, between the runway centerline extended and the glide path to the runway end is

$$\theta_{ra} = \tan^{-1}\left(\frac{y_t}{x_t}\right) = \tan^{-1}\left(\frac{646.6}{3815.5}\right) = \tan^{-1}(0.16947) = 9.62^\circ \tag{27}$$

*Note that the result in terms of C_L is the same as for $V^2/g\tan\phi = (105.6)^2/((32.174)(1)) = 346.5\,\text{ft}$

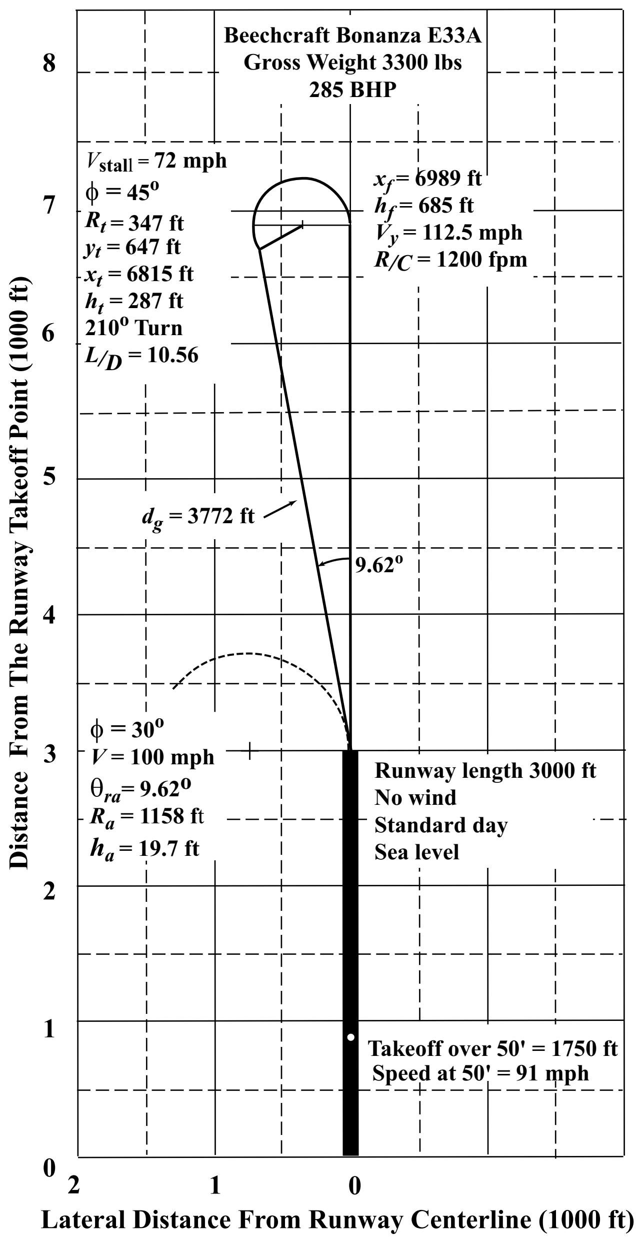

Figure 4. Scaled turnback diagram for an E33A Bonanza.

The geometry for the aircraft altitude after completing the turnback turn is shown in Fig. 6.

The aircraft is now a glider. The flight path angle, γ_g, is now negative and simply

$$\gamma_g = -\tan^{-1}\left(\frac{1}{L/D_{\max}}\right) = -\tan^{-1}\left(\frac{1}{10.56}\right) = -\tan^{-1}(0.094697) = -5.41° \tag{28}$$

where 10.56 is $L/D_{\max}$ for an E33A Bonanza in the glide. Henceforth, the glide path angle may be indicated as a positive number while understanding that it is, in fact, negative. Note that the glide path angle, $\gamma_g = 5.41°$, is less than the climb angle, $\gamma_c = 6.96°$ (Eq. 21). Thus, the flight path climb angle to flight path glide angle ratio $\gamma_c/\gamma_g > 1$. The available glide range for an E33A, where both h_f and h_t are positive numbers, is then

$$R_g = (h_f - h_t)L/D_{\max} = (685 - 287.4)(10.56) = (397.6)(10.56) = 4198.7\,\text{ft} \tag{29}$$

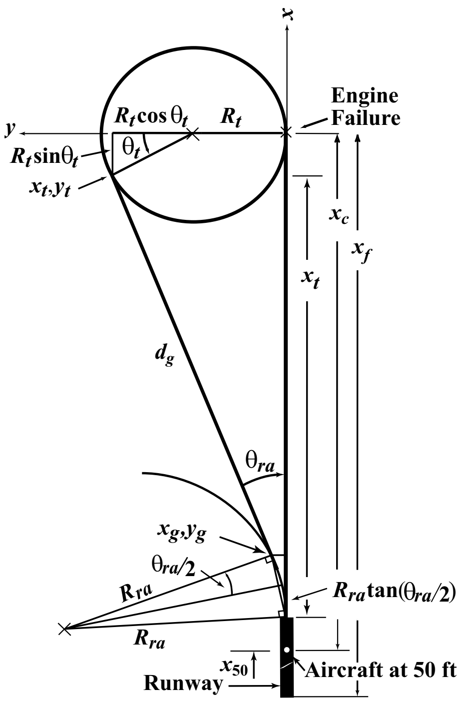

Figure 5. Detailed, not to scale turnback, plan view diagram when the aircraft heads for the departure end of runway.

The glide distance from the end of the 210° turnback turn to the tangent to a 100 mph 30° bank realignment turn to the end of a 3000 ft runway for an E33A climbing out at V_y is

$$d_g = \sqrt{(x_t - x_g)^2 + (y_t - y_g)^2} = \sqrt{(3815.5.7 - 96.1)^2 + (646.7 - 16.3)^2} = 3772.4\,\text{ft} \tag{30}$$

Here, from Fig. 5 (x_g, y_g), with $x_g = R_{ra}\tan(\theta_{ra}/2)\cos\theta_{ra}$ and $y_g = R_{ra}\tan(\theta_{ra}/2)\sin\theta_{ra}$, represents the tangent point on the realignment turn.

Comparing the results of Eq. (29) and Eq. (30), i.e., $R_g = 4198.7$ ft and $d_g = 3772.4$ ft, it is clear that an E33A can glide to the departure end of a 3000 ft runway.

The altitude lost during the glide from the end of the 210° turnback turn to a 100 mph 30° bank realignment turn is

$$h_g = \frac{d_g}{L/D_{\max}} = \frac{3660.8}{10.56} = 357.2.7\,\text{ft} \tag{31}$$

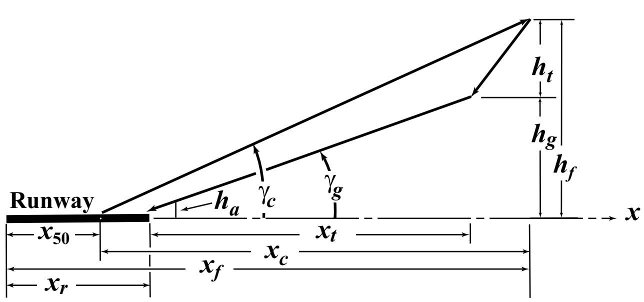

Figure 6. Detailed, not to scale, turnback altitude diagram.

However, the altitude lost during the realignment turn to the departure end of the runway must also be taken into account. In a 100 mph 30° bank turn the aircraft loses an additional altitude $h_a = 19.7$ ft. Thus, the total loss in altitude is

$$h_{total} = h_t + h_g + h_a = 287.4 + 357.2 + 19.7 = 664.3\,\text{ft} \tag{32}$$

which from a failure altitude of 685 ft leaves a cushion of 20.7 ft to allow for gear and flap extension as well as imperfect pilot execution.

E33A LONGER RUNWAYS

If the aircraft departs from a much longer runway *and* the pilot heads for the departure end of the runway, then the altitude loss in the turnback turn remains the same. The distance from the departure end of the longer runway is reduced. Hence, the glide distance to the tangent to the realignment turn is reduced as is the altitude loss. However, because the aircraft is closer to the departure end of the runway, the realignment angle increases. Thus, the altitude loss in the realignment turn also increases. Consequently, there is a trade-off between the reduction in altitude loss in the glide and the increase in altitude loss in the realignment turn.

Let's look at a longer runway, e.g., 4000 ft, from the same 685 ft failure altitude and turnback turn of 210°. In this case, the realignment angle increases to 12.94°. Because of the reduced glide distance and altitude loss in the glide (2757.5 ft and 261.1 ft) which is somewhat offset by the increase in altitude loss in the 12.94° realignment turn (26.5 ft) the available altitude cushion increases to 110 ft.

However, if the runway length is quite long, say 6000 ft, and the pilot turns back through two hundred seventy degrees in a 45° bank at $C_{L\max}$ the altitude loss in the turnback turn is 369.5 ft. Here, at the end of the turnback turn, the axial distance from the runway end is, $x_t = 642.2$ ft. Furthermore, the lateral offset to the runway end is $y_t = 346.6$ ft, i.e., the radius of the turnback turn. The departure end of the runway now bears approximately 45° from the aircraft. The required realignment turn angle to align the aircraft with the runway is effectively ninety degrees. There is no glide distance. Hence, the realignment turn must be conducted using a 45° bank angle and at $C_{L\max}$ to make the departure end of the runway. Any lower bank angle and/or higher turn speed 'overshoots' the runway centerline. The altitude loss in the realignment turn is approximately 123.1 ft. The total altitude loss is 492.6 ft.

This is clearly Case 2, Circle 5 (see Fig. 1c), as discussed above. Notice that the failure distance, x_f, for an E33A climbing out at V_y is 988 ft from the departure end of the 6000 ft runway. Hence, the failure distance is approximately three radii ($2.85\,R_t$) from the departure end of the 6000 ft runway. Also, notice that, from a failure altitude of 685 ft, the aircraft is at nearly two hundred feet (192.4 ft) over the runway centerline at the end of the maneuver. Although, this maneuver may be considered a success, it is very much an edge of the envelope maneuver. In this case, the pilot is better off heading directly for a point between the takeoff and departure ends of the runway i.e., Case 3 above. Let's look at this.

DIRECT HEADING CASE

On a standard day at sea level at gross weight gear down and full flaps, the POH for the 1969 E33A gives the landing distance over a 50 ft obstacle as 1150 ft and the ground run as 625 ft. If the pilot executes a 'classic' turnback turn from 685 ft through 210°, the aircraft loses 287.4 ft of altitude. At the end of the turnback turn the aircraft position is $(x_t, y_t) = (815.5, 646.7)$ ft with a heading of 30° relative to the runway. At the end of the turnback turn, the aircraft altitude is 397.6 ft. Suppose that the pilot maintains heading at the completion of the turnback turn direct to the intersection with the runway.

The questions are:
where does the direct heading flight path intersect the runway;
at what altitude does the intersection occur;
is there sufficient altitude to effect a realignment turn;
is there sufficient runway to successfully land the aircraft?

At the end of the 210° turnback turn a direct heading of 30° relative to the runway centerline yields an intersection with the runway, x_i

$$x_i = x_t + x_r - \frac{y_t}{\tan(\theta_t - 180)} = 815.5 + 6000 - 1120.2 = 5695.3\,\text{ft} \tag{33}$$

from the takeoff end of the runway.

From the geometry in Fig. 5, the glide distance to a the tangent to a 100 mph, 30° bank realignment turn is

$$d_g = \sqrt{(x_t - x_g)^2 + (y_t - y_g)^2} = 983.2\,\text{ft} \tag{34}$$

The altitude loss in the 983.2 ft glide is 93.1 ft. The altitude loss in the realignment turn is 61.4 ft. The aircraft altitude over the runway is 243.1 ft. It is also 50.7 ft higher than in the Case 2 Circle 5 example above. With the aircraft aligned with the runway, this altitude is more than adequate to configure the aircraft for landing and to land and stop on the remaining runway. In fact, the pilot may also have to execute S-turns and/or slip the aircraft to lose sufficient altitude to land and stop on the runway. The maneuver is considered a success. Furthermore, in terms of cushion the 'classic' turnback maneuver is more optimal.

For an E33A the climb flight path angle, γ_c, is larger than the glide flight path angle, γ_g. What if the glide flight path angle is larger than the climb flight path angle, i.e., $\gamma_g > \gamma_c$? This is the case for the 1974 Cessna 172M

1974 Cessna 172M

Turning now to a 150 BHP Cessna 172M the POH gives the velocity for maximum rate of climb, $V_{R/C\text{max}}$ or V_y, as 91 mph or 133.5 ft/sec while the maximum rate-of-climb, R/C_{max}, is 645 ft/min or 10.75 ft/sec. Again, from Fig. (2) the climb flight path angle, γ_c, is

$$\gamma_c = \sin^{-1}\left(\frac{R/C_{\text{max}}}{V_{R/C\text{max}}}\right) = \sin^{-1}\left(\frac{10.75}{133.5}\right) = 4.6^\circ \tag{35}$$

which is considerably smaller than for the E33A (4.6° vs 6.96°).

Again, using the time, t_c, it takes a C172M to climb from 50 ft to the failure altitude h_f, at the speed for maximum rate of climb, $V_{R/C\text{max}}$, the distance from 50 ft, x_c, is

$$x_c = \frac{h_c}{R/C_{\text{max}}} V_{R/C\text{max}} = \frac{485 - 50}{10.75} 133.5 = (40.47)(133.5) = 5402.7\,\text{ft} \tag{36}$$

Here, the failure altitude, h_f, for the C172M is assumed to be 485 ft AGL. From the C172M POH the takeoff distance over a 50 ft obstacle is $x_{50} = 1525$ ft. Thus, it takes 40.47 seconds to climb from 50 ft to 485 ft. The resulting total estimated ground distance from the beginning of the takeoff run is thus $x = 6925.7$ ft (5402.7+1525). Notice that the C172M climb distance, x_c, is approximately the same as that for the E33A (5239 vs 5402 ft), although the assumed failure altitude is 200 ft less for the C172M than for the E33A Bonanza. This results because of the smaller climb angle, γ_c, for the C172M. The climb angle is lower for the C172 than for the E33A (4.6° vs 6.9°) because of the 'higher' power loading, (W/BHP), of 15.3 for the C172M compared with 11.6 for the E33A.

Again, using the simple geometry shown in Fig. 5 the C172M glide distance to the end of the runway and the realignment turn angle can be estimated. First, the radius of the unpowered gliding turn back to the runway end at $C_{L\text{max}}$ in a 45° bank is simply

$$R_t = \frac{1}{g\tan\phi}\,\frac{2}{\sigma\rho_{\text{SSL}}}\,\frac{W}{S}\,\frac{1}{C_{L\text{max}}} = \frac{1}{(32.174)(1)}\,\frac{2}{0.002377}\,\frac{2300}{174}\,\frac{1}{1.5914} = 217.2\,\text{ft} \tag{37}$$

From Fig. 5 the lateral offset from the extended runway centerline, y_t, for a 210° turnback turn in the C172M is

$$y_t = R_t(1 + \cos(30^\circ)) = (217.2)(1 + 0.866) = 405.3\,\text{ft} \tag{38}$$

The distance from the departure end of a 3000 ft runway if the C172M climbs at V_y is

$$x_t = x_c + x_{50} - x_r - R_t\sin(30^\circ) = 5402.7 + 1525 - 3000 - (217.2)(0.5) = 3819.1\,\text{ft} \tag{39}$$

The realignment angle, θ_{ra}, between the runway centerline extended and the glide path to the runway end is

$$\theta_{ra} = \tan^{-1}\left(\frac{y_t}{x_t}\right) = \tan^{-1}\left(\frac{405.3}{3819.1}\right) = \tan^{-1}(0.106124) = 6.06^\circ \tag{40}$$

The C172M is now a glider. The flight path angle, γ_g, is now negative and is

$$\gamma_g = -\tan^{-1}\left(\frac{1}{L/D_{\text{max}}}\right) = -\tan^{-1}\left(\frac{1}{9.2}\right) = -\tan^{-1}(0.108696) = -6.2^\circ \tag{41}$$

where 9.2 is $V_{L/D\text{max}}$ for a Cessna C172M.

The available glide range for a C172M, where both h_f and h_t are positive numbers, is then

$$R_g = (h_f - h_t)L/D_{\text{max}} = (485 - 196.4)(9.2) = (288.6)(9.2) = 2655.1\,\text{ft} \tag{42}$$

The glide distance from the end of the 210° turnback turn to the tangent to a 70 mph 30° bank realignment turn to the end of a 3000 ft runway for a C172M climbing out at V_y is

$$d_g = \sqrt{(x_t - x_g)^2 + (y_t - y_g)^2} = \sqrt{(3819.1 - 34.3)^2 + (405.3 - 3.6)^2} = 3804.1\,\text{ft} \tag{43}$$

Comparing the results of Eq. (42) and Eq. (43), i.e., $R_g = 2655.1$ ft and $d_g = 3804.1$ ft, it is clear that a C172M **cannot** glide to the departure end of a 3000 ft runway from a failure altitude of 485 ft given that the required glide distance, d_g, exceeds the available glide range, R_g by more than a thousand feet.

Furthermore, looking at the altitude lost during the glide results in

$$h_g = \frac{d_g}{L/D_{\text{max}}} = \frac{3804.1}{9.2} = 413.5\,\text{ft} \tag{44}$$

In addition, the altitude loss during a realignment turn at the POH's recommended approach speed of 70 mph to the departure end of the runway in a 30° bank turn is

$$\begin{aligned} h_a &= \left[\frac{C_D}{C_L^2}\frac{4W}{\rho S g}\frac{1}{\sin 2\phi}\right]\Delta\Psi = \left[\frac{0.1078}{(1.055)^2}\frac{4(2300)}{0.002377(174)(32.174)}\frac{1}{\sin(60)}\right](6.83/57.296) \\ &= (0.09685)(691.4)(1.155)(0.1192) \\ &= 9.2\,\text{ft} \end{aligned} \tag{45}$$

Thus, the total altitude loss in the maneuver is

$$h_{total} = h_t + h_g + h_a = 194.6 + 413.5 + 9.2 = 617.3\,\text{ft} \tag{46}$$

which also clearly indicates that a C172M **cannot** execute a successful turnback maneuver to the end of a 3000 ft runway from a failure altitude of 485 ft $(485 - 617.3) = -132.3$ ft).

Fundamentally, this is because the climb flight path angle, $\gamma_c = 4.6^\circ$ (Eq. 35), is smaller than the glide angle $\gamma_g = 6.2^\circ$ (Eq. 41). As a result, the available glide range is less than the direct distance to the departure end of the runway.

C172M A Longer Runway

What if the runway is longer? Let's add 50% to the length of the runway. Here, everything remains the same except (x_t, y_t), is now equal to $(2317.1, 405.3)$, and the realignment angle, θ_{ra}, is now equal to 9.92°. Here, the glide distance, d_g, from (x_t, y_t) to the tangent to the realignment turn, (x_g, y_g) is

$$\begin{aligned} x_g &= R_{ra}\tan(\theta_{ra}/2)\cos\theta_{ra} = 651.4\tan(4.96)\cos(9.92) = 55.7\,\text{ft} \\ y_g &= R_{ra}\tan(\theta_{ra}/2)\sin\theta_{ra} = 651.4\tan(4.96)\sin(9.92) = 9.7\,\text{ft} \end{aligned} \tag{47}$$

From Eq. (47) the glide distance to the realignment turn to a 4500 ft runway for a C172M climbing out a V_y is now

$$d_g = \sqrt{(x_t - x_g)^2 + (y_t - y_g)^2} = \sqrt{(2261.4)^2 + (395.6)^2} = 2295.8\,\text{ft}$$

The altitude lost in the glide to the 4500 ft runway realignment turn is now

$$h_g = \frac{d_g}{L/D_{\max}} = \frac{2295.8}{9.2} = 249.5\,\text{ft} \tag{48}$$

The realignment turn through an angle of 9.92° results in an altitude loss of 15 ft. Thus, the total altitude lost during the turnback to a 4500 ft runway from a 450 ft failure altitude is

$$h_{total} = h_t + h_g + h_a = 194.6 + 249.5 + 15 = 459.1\,\text{ft} \tag{49}$$

which leaves a cushion of 25.9 ft for flap extension and landing flare.

The simple conclusion here is that aircraft with higher power loadings, W/BHP, and larger gliding flight path angles, e.g., the C172M, likely require relatively longer runways than aircraft with smaller power loadings and smaller gliding flight path angles, e.g., the E33A. Another way of saying this is 'the runway end needs to be brought to the aircraft'.

A Comparison

Figure 7 compares the minimum failure altitude, h_f, for an E33A and a Cessna 172M versus runway length when the aircraft heads for the departure end of the runway after a 210° turnback turn. The cushion value is 20.01 ft.

Notice that the minimum runway length for the E33A decreases as the failure altitude increases. This is illustrated by the negative slope (-0.3858) in the linear curve fit to the six highest h_f calculated points in Fig. 7a. It is also illustrated by the negative first order term, x_r, in the second degree fit in Fig. 7a. For example: for a 2500 ft runway length the minimum failure altitude is approximately 875 ft, while for a 3200 ft runway the minimum failure altitude is approximately 615 ft.

For the C172M the minimum failure altitude, h_f, increases linearly with runway length as indicated by the positive slope (0.3562) in Fig. 7b. In particular, the minimum h_f increases from 485 ft for a runway length of approximately 4445 ft to over 6020 ft for a minimum h_f of 1000 ft.

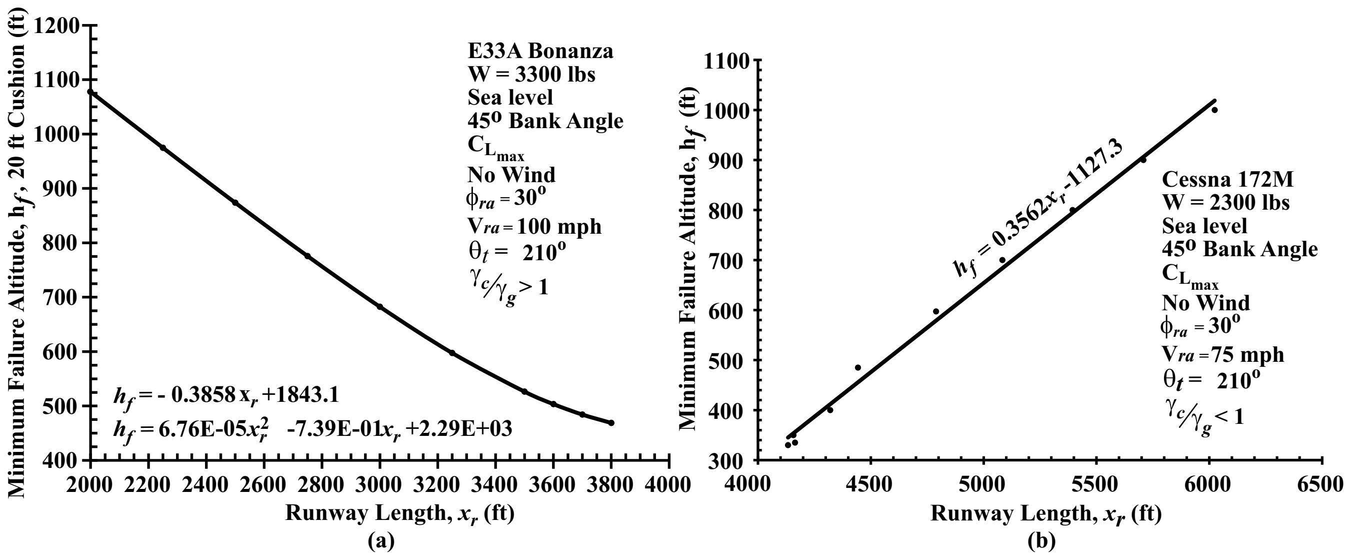

Figure 7. Comparison of the minimum failure altitude for an E33A and a C172M when the aircraft heads for the runway end.

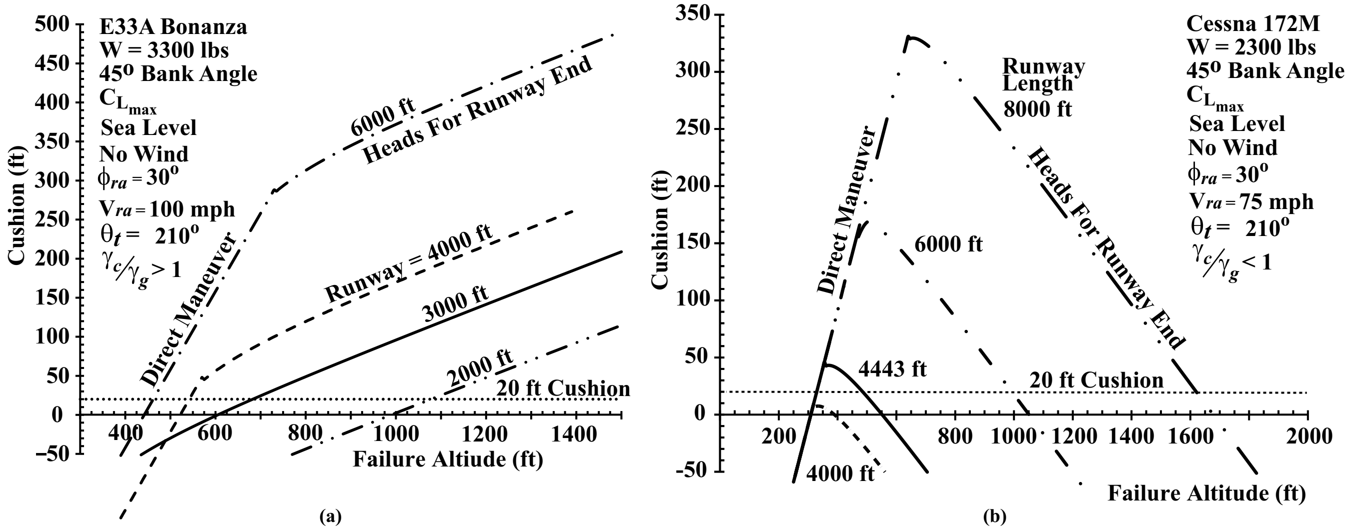

Figure 8. Comparison using 20 ft cushions for an E33A and a Cessna 172M for various runway lengths.

Figure 8 is an alternate way of looking at the results shown in Fig. 7. Figure 8 compares the aircraft cushion versus failure altitude for the E33A Beechcraft Bonanza and the Cessna C172M for various runway lengths. A 20 ft cushion is assumed. The ability to maneuver the aircraft to land on the remaining runway once aligned with the runway is also assumed. Figure 8a shows that, beyond a certain minimum failure altitude, the E33A can execute a successful turnback maneuver from any reasonable failure altitude.

The bend in the 4000 ft and 6000 ft curves in Fig. 8a indicates the failure altitude beyond which the aircraft heads for the end of the runway. Prior to that point failure occurs either before the departure end of the runway or within four radii beyond the departure end of the runway. In that case, a more optimal Direct maneuver is indicated. Note that both prior to the bend in the curve and after, the available cushion varies linearly with failure altitude.

For the 2000 ft runway a Direct maneuver is indicated for failure altitudes from approximately 200-245 ft. These are not shown in Fig. 8a because the cushion is negative. Similarly, for the 3000 ft runway a Direct maneuver is indicated for failure altitudes from approximately 200-682 ft. However, the cushion is either negative or less that 20 ft. A negative cushion or one less than 20 ft is considered an unsuccessful turnback maneuver.

Turning to the C172M, Fig. 8b illustrates that the aircraft can only execute a successful turnback maneuver within a specific range of failure altitudes. In Fig. 8b, that range of failure altitudes is represented by the intersection of the specific altitude curves and the 20 ft cushion line.

Figure 8b illustrates that a C172M cannot successfully complete a turnback maneuver with a 20 ft cushion to a 4000 ft runway. Here, the C172M does not have enough altitude after the completion of the turnback turn to execute the glide on a 210° heading Direct to the tangent to the realignment turn and the realignment turn itself with a 20 ft cushion.

For a 4443 ft runway, Fig. 8b shows that a C172M can successfully complete a turnback maneuver from failure altitudes of approximately 330 ft to 485 ft. The maximum point of the curves indicates the approximate change over point from a Direct turnback maneuver to a classical 210° turnback maneuver to the end of the departure runway. Up to a failure altitude of approximately 355 ft failure occurs before the departure end or before four radii from the departure end of the runway. Hence, a Direct turnback maneuver is indicated. Beyond four radii the aircraft returns to the departure end of the runway. Figure 8b also shows similar limited failure altitude ranges for successful turnback maneuvers for 6000 ft and 8000 ft runways.

The turnback maneuvers beyond the upper end of the permissible range are unsuccessful because the C172M's flight path glide angle to flight path climb angle, γ_g/γ_c, is less than one. Upon completion of the initial turnback turn the C172M simply does not have enough glide range to glide to the departure end of the runway with sufficient excess altitude to complete the realignment turn with a 20 ft cushion.

With a $\gamma_c/\gamma_g > 1$ the E33A can glide to a shorter runway than the C172M with a $\gamma_c/\gamma_g < 1$. Arbitrarily increasing the failure altitude for aircraft with γ_c/γ_g ratios less than one may result in the inability of the aircraft to glide to the end of the runway upon completing the turnback turn.

Clearly, these two examples indicate that the flight path glide angle to flight path climb angle ratio, γ_g/γ_c, is a governing parameter that characterizes the turnback maneuver.

Aeronca 7AC

Turning now to a 1946 65 BHP Aeronca 7AC, the POH gives the velocity for maximum rate of climb, $V_{R/C\text{max}}$, as 60 mph or 88 ft/sec, while the maximum rate-of-climb, R/C_{max}, is just 370 ft/min or 6.17 ft/sec. Again, from Fig. (3) the climb flight path angle, γ_c, is

$$\gamma_c = \sin^{-1}\left(\frac{R/C_{\text{max}}}{V_{R/C\text{max}}}\right) = \sin^{-1}\left(\frac{6.17}{88}\right) = 4.02^\circ \tag{50}$$

which is considerably smaller than for the E33A (6.96°) and somewhat smaller than for the C172M (4.6°).

For the 7AC, initially assume a failure altitude of 250 ft. Using the same techniques as for the E33A and for the C172M, the time it takes a 7AC to climb from 50 ft to the failure altitude, h_f, at the speed for maximum rate of climb, $V_{R/C\text{max}}$, is 32.41 (200/6.17) seconds. The distance from 50 ft to 250 ft, x_c is then

$$x_c = \frac{h_c}{R/C_{\text{max}}} V_{R/C\text{max}} = \frac{250-50}{6.17} 88 = (32.41)(88) = 2852.1\,\text{ft} \tag{51}$$

From the 7AC POH the takeoff distance over a 50 ft obstacle, x_{50}, is estimated to be 780 ft. The resulting total estimated ground distance from the beginning of the takeoff run is $x_f = 3632.1$ ft $(2852.1 + 780)$. Notice that the 7AC is closer to the runway end than either the E33A or the C172M. This is a result of the combination of a lower failure altitude and a lower climb speed even though the 7AC climb angle, γ_c, is smaller. Notice also that the times to climb from 50 ft to the failure altitude, h_f, for the E33A (30 sec.), the C172M (37.2 sec.) and the 7AC (32.41 sec.) are all approximately the same with an average time of 33.2 seconds.

Using Fig. 5, the data for the 7AC glide distance to the tangent to the realignment turn is calculated. First, the radius of the unpowered gliding turnback turn executed at $C_{L\text{max}}$ in a 45° bank is

$$R_t = \frac{1}{g\tan\phi}\,\frac{2}{\sigma\rho_{\text{SSL}}}\,\frac{W}{S}\,\frac{1}{C_{L\text{max}}} = \frac{1}{(32.174)(1)}\,\frac{2}{0.002377}\,\frac{1220}{170.22}\,\frac{1}{1.9414} = 96.55\,\text{ft} \tag{52}$$

which is less than half the C172M turn radius and almost three times smaller than the turn radius for the E33A.

From Fig. 5 the lateral offset from the extended runway centerline, y_t, for a 210° turnback turn is

$$y_t = R_t(1+\cos(30^\circ)) = (96.55)(1+0.866) = 180.2\,\text{ft} \tag{53}$$

If the 7AC climbs at V_y, the distance from the departure end of a 3000 ft runway is

$$x_t = x_c + x_{50} - x_r - R_t\sin(30^\circ) = 2852.1 + 780 - 3000 - (96.55)(0.5) = 584.2\,\text{ft} \tag{54}$$

The realignment angle, θ_{ra}, for the 7AC between the runway centerline extended and the glide path to the runway end is

$$\theta_{ra} = \tan^{-1}\left(\frac{y_t}{x_t}\right) = \tan^{-1}\left(\frac{180.2}{584.2}\right) = \tan^{-1}(0.30846) = 17.14^\circ \tag{55}$$

The flight path angle, γ_g, during the 7AC's glide is negative. Using the 7ACs POH value for $L/D_{\max}$, the 7ACs glide angle is

$$\gamma_g = -\tan^{-1}\left(\frac{1}{L/D_{\max}}\right) = -\tan^{-1}\left(\frac{1}{8.78}\right) = -\tan^{-1}(0.113895) = -6.5^\circ \tag{56}$$

The 7AC available glide range, where both h_f and h_t (see Eq. 20) are positive numbers, is then

$$R_g = (h_f - h_t)L/D_{\max} = (250 - 116.6)(8.78) = 1171.3\,\text{ft} \tag{57}$$

The glide distance from the end of the 210° turnback turn to the tangent to a 60 mph 30° bank realignment turn to the end of a 3000 ft runway for a 7AC climbing out at V_y is

$$d_g = \sqrt{(x_t - x_g)^2 + (y_t - y_g)^2} = \sqrt{(584.2 - 59.9)^2 + (180.2 - 18.4)^2} = 548.7\,\text{ft} \tag{58}$$

Comparing the results of Eq. (57) and Eq. (58), i.e., $R_g = 1171.3$ ft and $d_g = 548.7$ ft, it is clear that the 7AC *can* glide to the departure end of a 3000 ft runway.

Furthermore, looking at the altitude lost during the glide, h_g, is simply

$$h_g = \frac{548.7}{L/D_{\max}} = \frac{548.7}{8.78} = 62.5\,\text{ft} \tag{59}$$

At a speed of 60 mph in a 30° bank, the 7AC loses an additional altitude, $h_a = 18.8$ ft, in the realignment turn. Thus, recalling that for the 7AC the altitude loss in the 210° turn, Eq. (20), is 116.6 ft, the total loss in altitude in the maneuver is

$$h_{total} = h_t + h_g + h_a = 116.6 + 62.5 + 18.9 = 198\,\text{ft} \tag{60}$$

for a cushion of 62 ft, which is more than adequate to allow a successful landing on the runway..

Notice that the magnitude of the the glide flight path angle, $\gamma_g = 6.5^\circ$, *exceeds* the magnitude of the climb flight path angle $\gamma_c = 4.2^\circ$. Based on the results for the E33A and C172M this suggests that a turnback should *not* be successful. A to scale drawing of the 7AC turnback from 250 ft to a 3000 ft runway is shown in Fig. 9.

A Long Runway - Aeronca 7AC

However, with a cushion of 62 ft for a failure altitude of 250 ft it is reasonable to assume a successful turnback can be accomplished at a lower failure altitude. Let's try some lower failure altitudes, say in decrements of 10 ft.

All the failure altitudes down to 220 ft are successful. However, the realignment turn angle, θ_{ra} increases to 48.8° for a failure altitude of 220 ft. As the failure altitude decreases, the distance from the departure end of the runway also decreases. For a failure altitude of 210 ft the distance from the takeoff end of the 3000 ft runway is 3063 ft or 63 ft beyond the departure end of the runway. In addition, at the end of the 210° turnback turn the values for (x_t, y_t) are (15, 180.2) ft. Furthermore, for a failure altitude of 210 ft, the departure end of the 3000 ft runway bears approximately 71°. Thus, at the end of the 210° turnback turn the aircraft is abeam the departure end of the 3000 ft runway ($x_t = 15$ ft). If the pilot attempts to head for the departure end of the runway, a further turn of more than 60° is required which places the aircraft beyond the departure end of the runway. A further 90° turn followed by a reverse 45°, $C_{L_{\max}}$ turn to realign the aircraft with the runway centerline is required. Effectively, this is Case 4 above. The total altitude loss in the resulting 360°s of turning is 200 ft. A failure altitude of 210 ft is not considered successful because the cushion is less than 20 ft.

One might also consider increasing the failure altitude. If the failure altitude is increased, the aircraft is further from the departure end of the 3000 ft runway. Thus, the realignment angle is decreased. Again, let's increase the failure altitude in increments of 10 ft. The maneuver is successful up to a failure altitude of 315 ft. However, the cushion becomes unacceptable, i.e., less than 20 ft, above a failure altitude greater than 315 ft. The reason is that the available glide distance is insufficient to reach the departure end of the 3000 ft runway with an 'acceptable' cushion.

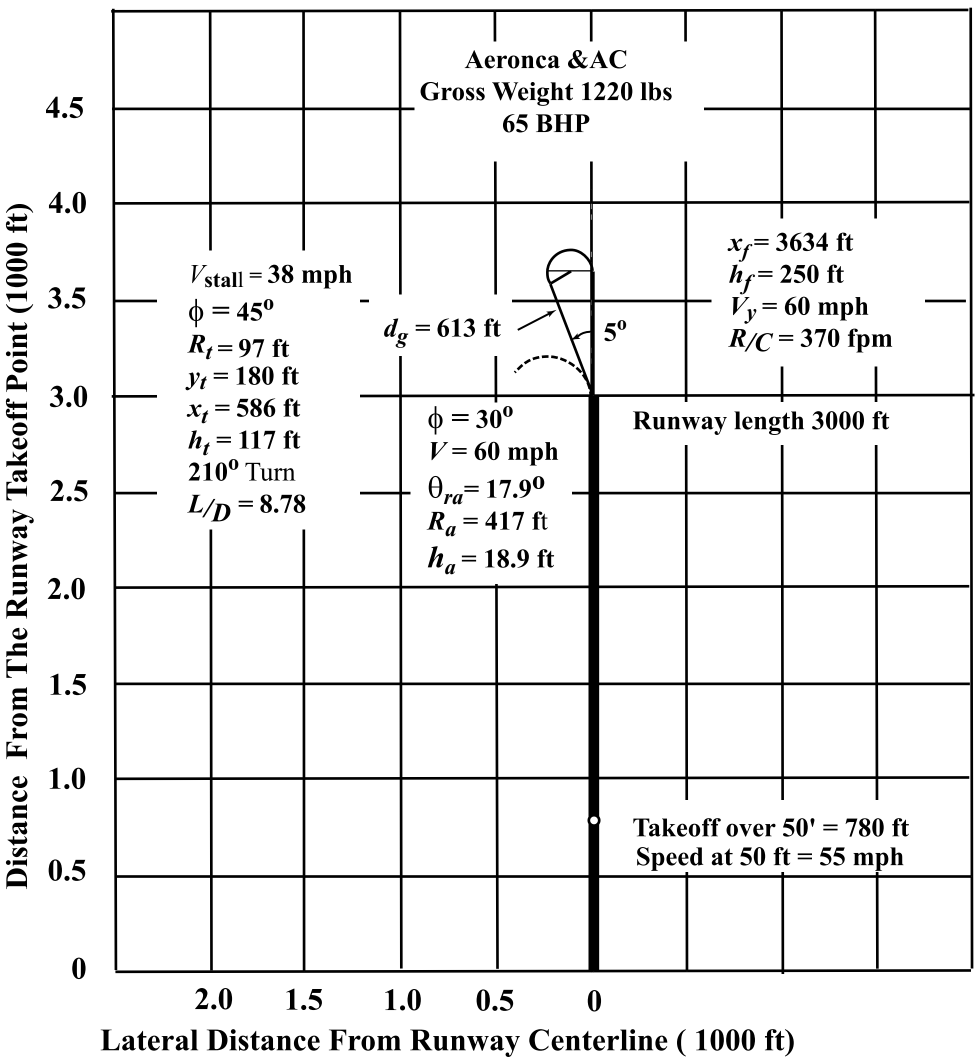

Figure 9. Scaled turnback diagram for an Aeronca 7AC: failure altitude 250 ft runway length 3000 ft.

Incrementally increasing the turnback angle above 210° while keeping the 315 ft failure altitude constant results in an unacceptable cushion. Furthermore, from a failure altitude of 210 ft decreasing the turnback angle below 199° also results in an unacceptable cushion.

These results suggest that, if the magnitude of $\gamma_g > \gamma_c$ and the runway is 'long' relative to the takeoff ground roll, the result may be a narrow band of successful failure altitudes *if*, on completion of the turnback turn, the pilot heads for the departure end of the runway. In this case, given that the landing ground roll is typically less than the takeoff ground roll, the pilot might be wise to head for an intermediate point on the runway sufficient for landing, i.e., a Direct maneuver, as suggested by Fig. 1.

Figure 10 shows three different turnback scenarios for the Aeronca 7AC for a failure altitude of 210 ft. These three scenarios are clearly examples of Case 3 as discussed above. The thick solid black line shows the 'classic' 210° turn flown at $C_{L\max}$ in a 45° bank. The thick dashed line extending from the end of the 210° turnback turn direct to the end of the 3000 ft runway with *no* realignment turn, represents a 180 ft glide at $V_{L/D\max} = 60$ mph to the departure end of the 3000 ft runway. Notice that the angle between the dashed line and the runway centerline extended is 85.3°, i.e., nearly 90°. The altitude lost during that 180 ft glide direct to the departure end of the runway is 20.6 ft. The total altitude lost in the turnback turn maneuver and the glide to the departure end of the runway is 137.2 ft.

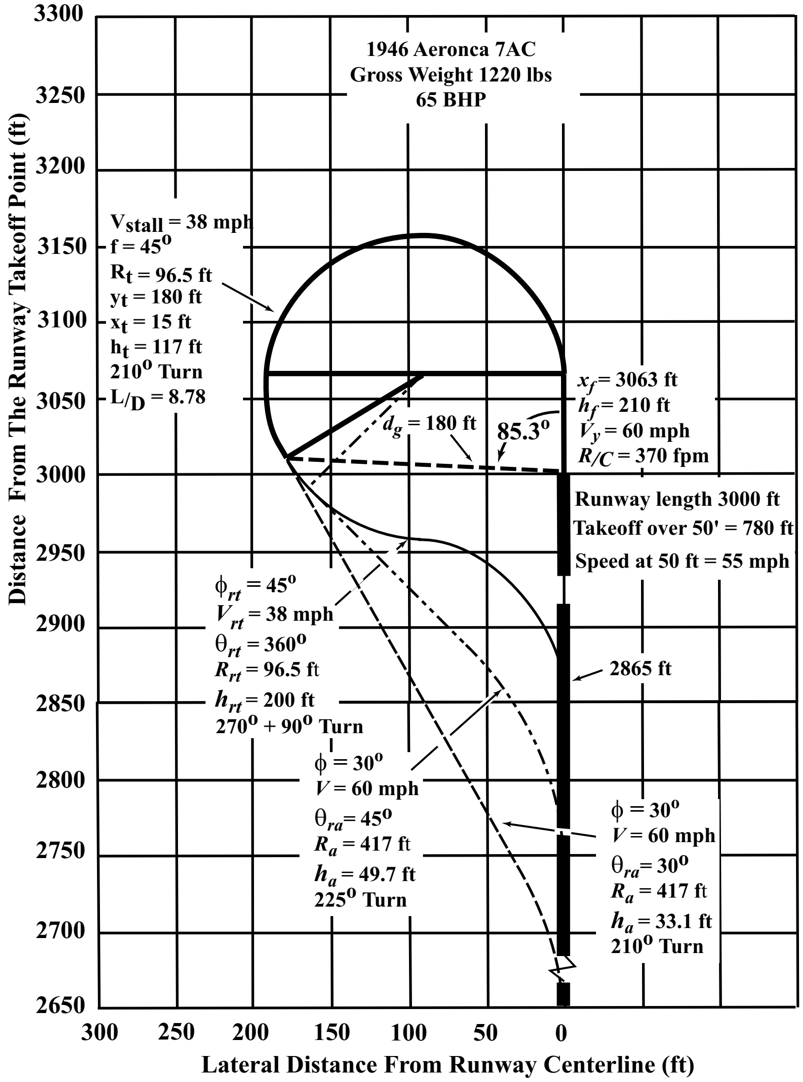

Figure 10. Scaled turnback diagram for an Aeronca 7AC: failure altitude 210 ft, runway length 3000 ft, 270° turn at $C_{L\max}$, bank angle 45° followed by a reverse 90° turn also at $C_{L\max}$ and a bank angle of 45°.

Thus, the aircraft arrives at the end of the departure runway at a altitude of 72.8 ft on a heading nearly 90° to the runway. Furthermore, without an additional heading change, the aircraft is *not* headed for the departure end of the runway. Clearly, Fig. 10 illustrates that this is *not* an acceptable solution.

However, three alternate maneuvers are shown in Fig. 10.

The thin solid line in Fig. 10 extending from the end of the thick 210° circle line illustrates the first alternate turnback maneuver. Here, the initial turn continues through 270° at $C_{L\max}$ in a 45° bank followed by an immediate reversal of the bank angle to a reverse ninety degree realignment turn. The reverse turn is also flown at a bank angle of 45° at $C_{L\max}$. Here, there is no glide segment. The total altitude loss in the turns is 200 ft. The aircraft is aligned with the runway at a point 2865 ft from the takeoff end of the 3000 ft runway at an altitude of 10 ft. This is an example of Case 3 Circle 4 in Fig. 1. Because of the small cushion, the maneuver is considered *marginally* successful.

Contributing to the success of the maneuver is the relatively long runway compared to the takeoff distance at fifty feet, the low stall speed which yields a small diameter for the turnback turn and executing the reverse turn at $C_{L\max}$ and in a 45° degree bank. This is definitely an edge of the envelope manuever.

The second alternate turnback maneuver represented by the chain dashed line in Fig. 10 turns through 225°, followed by a glide on a constant heading of 225° at $V_{L/D\max} = 60$ mph for an estimated distance of 60.4 ft to the tangent to a 417 ft radius realignment turn, i.e., a Direct maneuver. The altitude loss in the 225° turnback turn is 124.9 ft. The altitude loss in the short glide is an estimated 6.9 ft. The 30° bank 60 mph realignment turn costs an additional 49.7 ft. The total altitude lost during the maneuver is 181.5 ft. Hence, the aircraft is over the remaining runway at 28.5 ft at an estimated distance of 2657.5 ft from the takeoff end of the 3000 ft runway. This maneuver, a slightly modified 'classic' turnback maneuver called the Direct maneuver, is considered a success. The Direct maneuver takes advantage of the relatively long runway.

The third alternate turnback maneuver is represented by the thin dashed line in Fig. 10. Here, after a 210° turn, the aircraft continues gliding on a constant heading of 210° at $V_{L/D\max} = 60$ mph for an estimated 249 ft to the tangent to a realignment turn with a radius of 417 ft. The altitude loss during this glide is an estimated 28.3 ft. The altitude loss in the 30° bank angle 60 mph realignment turn is an estimated 28.3 ft. The total altitude loss is 178 ft. Thus, the aircraft is at an altitude of 32 ft over the runway at an estimated distance of 2591.2 ft from the takeoff end of the 3000 ft runway. The Direct maneuver is considered a success. Again, the Direct maneuver is a basic 'classic' turnback maneuver which takes advantage of the relatively long runway.

The second and third maneuvers discussed immediately above are modifications of Case 3 Circles 4, 3 and 2 shown in Fig. 1 by the chained dashed lines. These maneuvers substitute a constant heading glide for the reverse 90° realignment turn discussed above. They depend upon sufficient runway remaining to effect a successful landing. Furthermore, notice that in each case the total altitude loss is less than for the 270/90° turnback maneuver. Generally, an increased glide distance is less 'expensive' than increased turning.

The results above illustrate that in most cases the 'classical' 210° turnback turn followed by a glide to the runway and a reverse realignment turn is preferred to alternatives. The Direct maneuver is strongly suggested when the failure distance occurs within four times the turnback turn radius ($4R_t$) of the departure end of the runway or prior to the departure end of the runway.

Results

Here, let's look at the effect of failure altitude, runway length, and the ratio of aircraft maximum climb angle to maximum glide angle in more detail. In particular, the next few figures illustrate the relative contributions of altitude loss in the turnback turn, glide and realignment turn on the total altitude loss as well as on the cushion.

Effect of Failure Altitude and Runway Length on Altitude Loss

E33A Bonanza

Figure 11 shows the effect of failure altitude on altitude loss for a 3000 ft runway. The total altitude loss, the altitude loss in the glide, the altitude loss for the turnback turn and for the realignment turn for an E33A Bonanza are shown. Three turnback turn angles, θ_t, of 195°, 210° and 225° are illustrated for failure altitudes from 685 ft to 2100 ft. Also shown is the altitude cushion when the aircraft arrives over the runway. Clearly, as expected, the total altitude loss, glide loss and cushion increase linearly with the failure altitude. Also, as expected, the turnback turn altitude loss is constant for all turnback turn angles, θ_t.

Figure 11 shows that the total altitude loss results all have essentially the same slope with an average value of 0.7743, as shown by the equation in Fig. 11 for the combined 195°, 210° and 225° calculation sets. The slope for the glide altitude loss is similar. Because the slope is less than one, the aircraft has sufficient altitude and glide distance to land on the 3000 ft runway even as the failure altitude increases beyond the minimum 685 ft. For example: A failure altitude of 1035 ft has a total altitude loss of 931 ft while a failure altitude of 1235 ft has a total altitude loss of 1086 ft. Thus, an increase in the failure altitude of 200 ft results in a smaller total altitude loss of 155 ft, i.e., a *decrease* of 155 ft. Notice also that the intercept value for the equation is positive at 130.71.

However, the insert in Fig. 11 also shows that the realignment turn altitude loss has a power law variation. The nonlinear variation occurs because, as the failure altitude increases, the distance from the takeoff end of the runway also increases. Thus, the realignment angle decreases (see e.g., Figs. 4 or 9). Hence, the realignment altitude loss

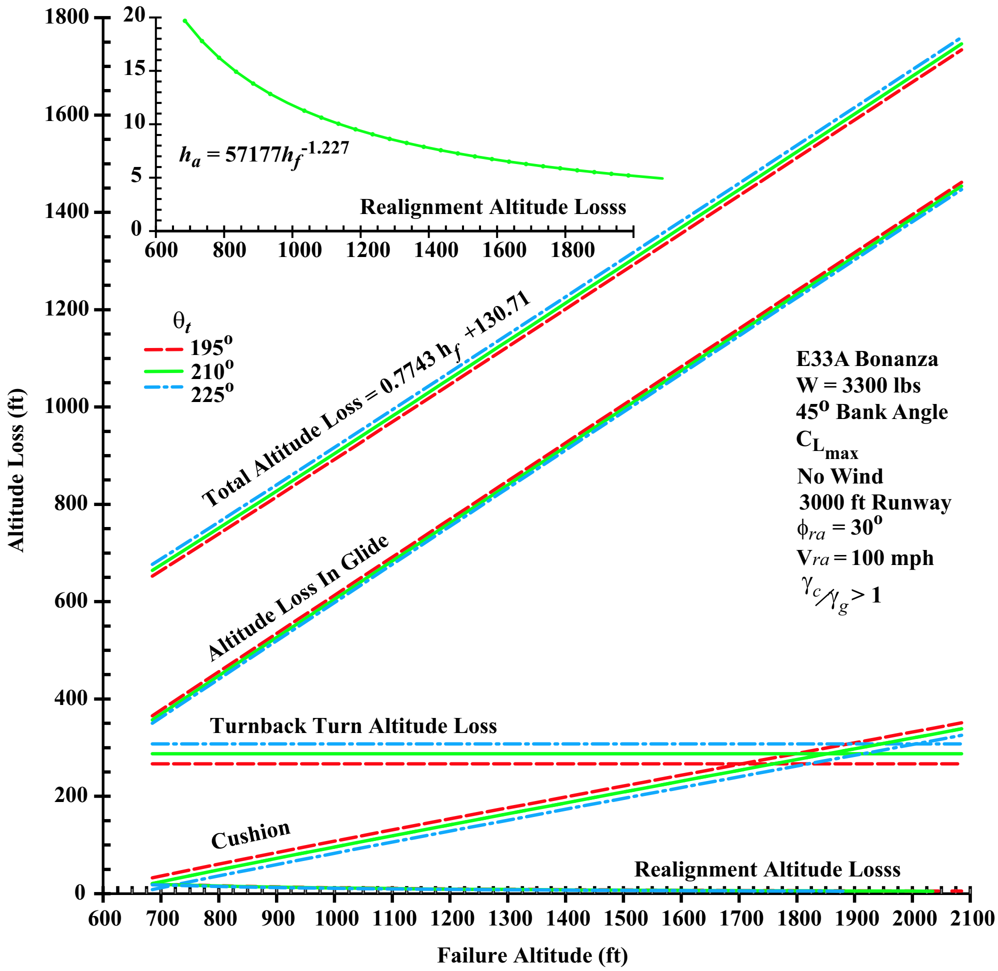

Figure 11. Altitude loss for an E33A Bonanza for various turnback turn angles for a 3000 ft runway.

also decreases but not linearly. Finally Fig. 11 shows that the altitude loss in the realignment turn is essentially negligible compared to the total altitude loss in the maneuver and to that lost in the turnback turn and the glide. This justifies the neglect of the altitude lost in the realignment turn in the original simplified model [2].

The altitude loss in the cushion also has a slope less than one. Hence, the cushion increases with increasing failure altitude. The increase in cushion with increasing altitude suggests that for any particular runway length, there is a minimum failure altitude equal to the sum of the altitude loss in the turnback turn and in the realignment turn, e.g., Case 5 (see Fig. 1c) above. The increase in cushion, i.e., altitude over the runway, may result in difficulties in landing the aircraft successfully on the runway.

Figure 12 illustrates the effect of increasing runway length for a fixed failure altitude of 685 ft for the E33A Bonanza. Here, the total and glide altitude losses decrease linearly with increasing runway length until the failure occurs within four turnback radii of the departure end of the runway. This occurs for an approximately 5600 ft runway for a 210° turnback turn to the end of the runway. In this region, the slope of the total altitude losses for turnback angles of 195°, 210° and 225° are essentially the same, as expected. Similarly, the increase in cushion with increasing runway length varies linearly. When the aircraft failure occurs before four times the turnback turn radius a Direct maneuver is more advantageous. Notice that for 210° the loss of total altitude line smoothly transitions into the constant altitude line. This is not the case for 195° and 225° turns.

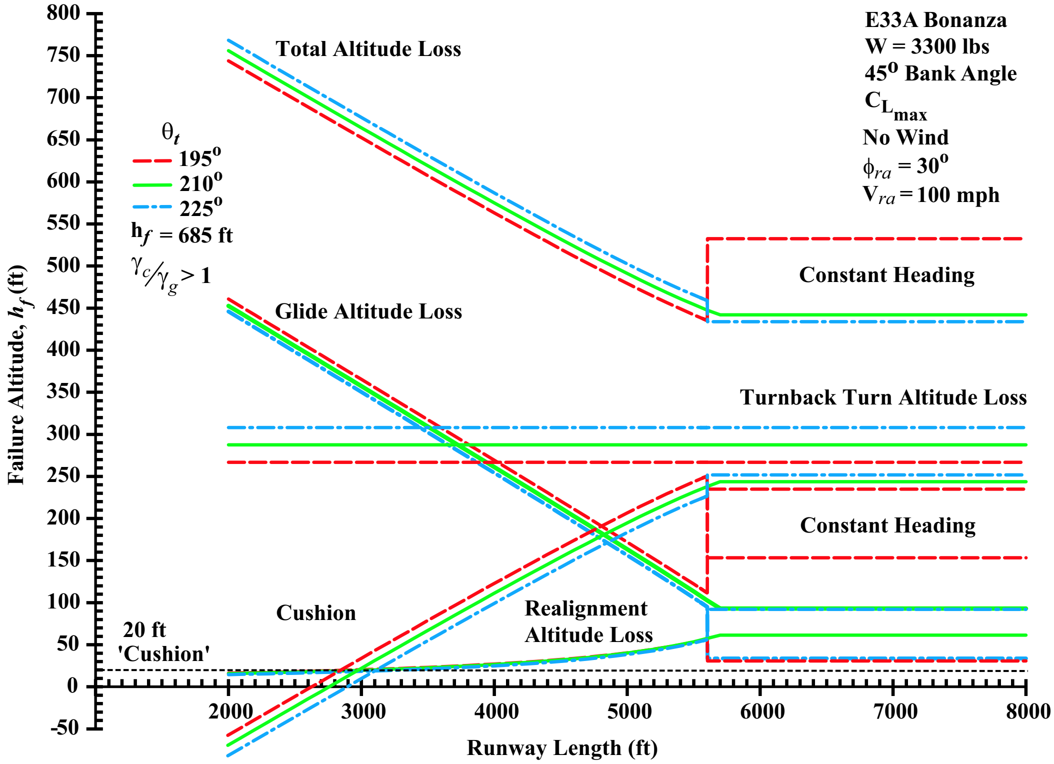

Figure 12. Altitude loss for an E33A Bonanza for various turnback turn angles and runway lengths for a 685 ft failure altitude.

Figure 12 also shows that the total altitude loss for a 195° Direct maneuver increases significantly. This is because the altitude lost in the increased glide distance to the runway is more than the reduction in altitude lost in the reduced turnback turn and the reduced realignment turn.

In the case of the 225° turnback turn, the increase in altitude loss in the Direct maneuver is offset by a decrease in glide distance to the runway. The result is a small decrease in total altitude loss.

For the E33A Figs. 11 and 12 illustrate the relative altitude loss for the turnback turn, the glide, and the realignment turn. For relatively short runway lengths, e.g., Fig. 11, for a 3000 ft runway, the altitude loss in the glide to the departure end of the runway exceeds the altitude loss in the actual turnback turn. However, for near minimum failure altitudes, Fig. 12 shows that for runways longer than approximately 3700 ft the altitude loss in the turnback turn exceeds that in the glide.

These observations suggest that a Direct heading close to the aircraft heading upon completion of the turnback turn is desirable. For the E33A, the optimal turnback turn is between 208° and 209°. Recall that airport runway designations are to the nearest ten degrees and aircraft directional gyros are similarly marked. Thus, a 210° turnback turn results in a 'smooth' transition to the Direct maneuver on a gyro cardinal heading. Thus, a 210° turnback maneuver followed by a Direct maneuver is a practical near 'optimal' turnback maneuver.

The optimal turnback turn Direct heading depends on aircraft performance, specifically the ratio of the flight path climb angle to the flight path glide angle, γ_c/γ_g, the bank angle and lift coefficient/speed at which the turnback turn is flown as well as the runway length.

Cessna 172M

Figure 13 shows the altitude loss for the Cessna 172M operating from a 4500 ft runway. Figure 13 looks very much like Fig. 11 but with significant differences. First, there appears to be only results for the 210° turnback angle. However, the results for the 195° and 225° turnback angles are represented by a cross (+) and by a dot • respectively.

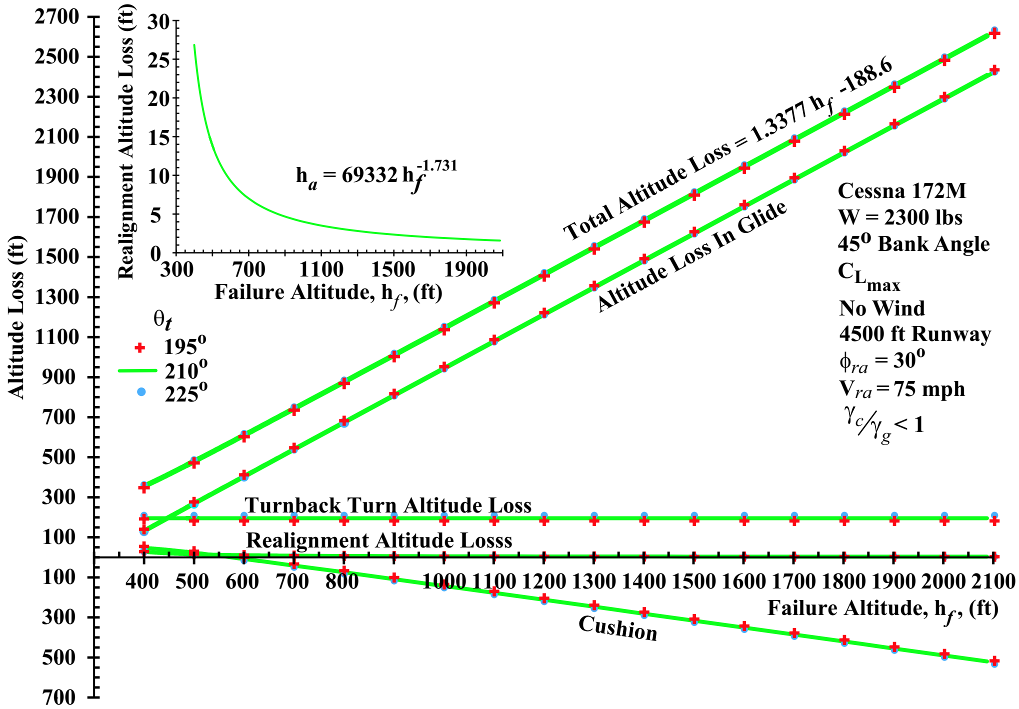

Figure 13. Altitude loss for a Cessna 172M for various turnback angles for a 4500 ft runway.

The most significant differences between Fig. 13 and Fig. 11 is that the slope of the total altitude line for the C172M is *greater* than one (1.3377) and the intercept is *negative* (-188.6). The greater than one slope and the negative intercept indicate that the total altitude loss exceeds the increase in failure altitude. Figure 11 for the E33A indicates that the slope of the total altitude line is *less* than one (0.7743) and the intercept is *positive* (+130.72). Both of these numbers show that for the E33A the total altitude loss is *less* than the *increase* in failure altitude (see also Fig. 7).

Furthermore, comparison of the 'Cushion' for the E33A (Fig. 11) and the C172M (Fig. 13) confirms the above observation (see also Fig. 8). Specifically, the 'Cushion' for the E33A increases with an increase in failure altitude while the 'Cushion' for the C172M decreases with increasing failure altitude. In fact, the 'Cushion' for the C172M becomes negative. The implication is that the E33A *can* return to a 3000 ft runway from any failure altitude above approximately 685 ft. However, the C172M *cannot* successfully return to a 4500 ft runway from a failure altitude greater than approximately 485 ft.*

Again, the reason is that the flight path climb angle to flight path glide angle ratio, γ_c/γ_g, is *greater* than one, ($\gamma_c/\gamma_g = 1.29$), for the E33A and *less* than one, ($\gamma_c/\gamma_g = 0.745$), for the Cessna 172M.

The inset in Fig. 13 shows that in comparison to the total altitude loss, the glide altitude loss and the turnback turn altitude loss, the power law altitude loss in the realignment turn is comparatively minor. Because of the low climb performance of the C172M at a failure altitude of 485 ft the aircraft is 6926 ft from the takeoff end of the 4500 ft runway. As a result, the realignment angle to the departure end of a 4500 ft is 9.9° degrees. The realignment altitude loss turning through 9.9° degrees is small (15 ft). Furthermore, if the C172M failure altitude is 1000 ft, the aircraft distance from the runway departure end is 8820 ft and the realignment angle is 2.7°. Again, the realignment altitude loss is quite small (4 ft). However, because of the altitude loss in the glide, the 'Cushion' for a turnback maneuver from a failure altitude of 1000 ft is -145 ft. The C172M simply cannot glide to the departure end of a 4500 ft runway from a failure altitude of 1000 ft.

*The actual numbers for an E33A are $h_f = 683$ ft and for the C172 $h_f = 505$ ft.

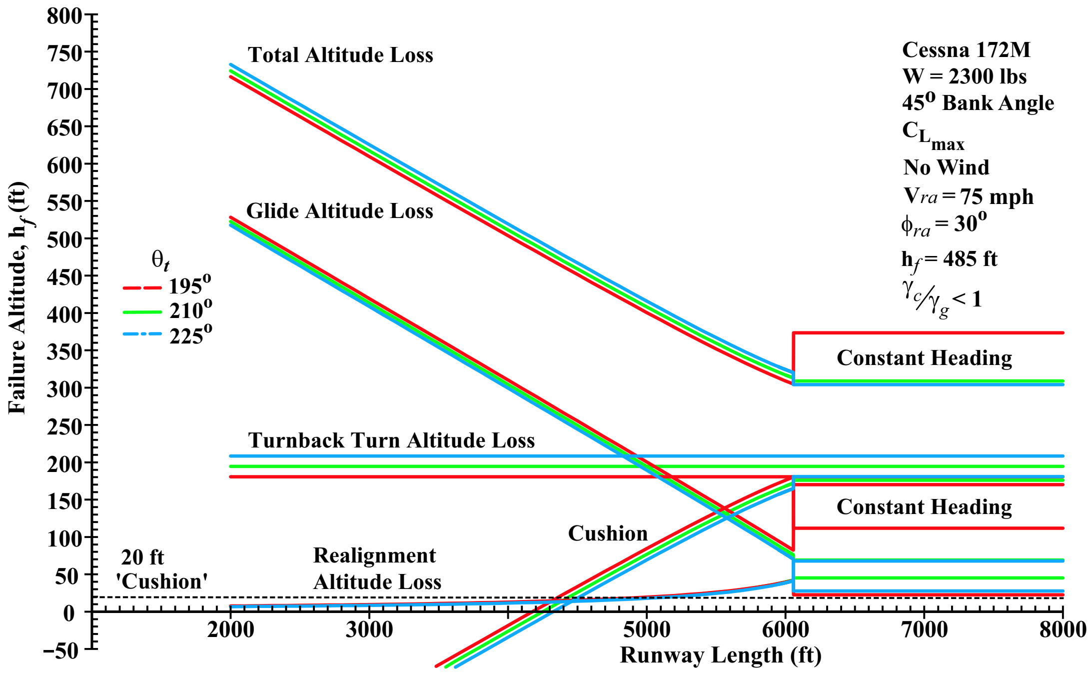

Figure 14. Altitude loss for a Cessna 172M for various turnback turn angles and runway lengths for a failure altitude of 485 ft.

Figure 14 for the Cessna C172M is again similar in form to Fig. 12 for the E33A Bonanza. Note that both graphs are presented at the same scale.

Recalling Figs. 11 and 13 and comparing Figs. 12 and 14, the runway length for a positive cushion is significantly longer for the C172M than for the E33A. This is an effect of the larger value of $\gamma_c/\gamma_g = 1.29$ for the E33A compared to that for the C172M of $\gamma_c/\gamma_g = 0.74$. Clearly, an aircraft with a climb angle to glide angle less than one requires a longer runway than an aircraft with a γ_c/γ_g greater than one.

For the E33A the Direct case occurs for runway lengths beyond approximately 5600 ft while for the C172M the Direct case occurs for runway lengths greater than approximately 6100 ft. Furthemore, notice that the cushion for the E33A is approximately 250 ft compared to a cushion of approximately 200 ft for the C172M for the Direct case. Considering that the E33A can add flaps and extend the gear to 'dirty' up the aircraft for descent while the C172M can only add flaps, it may be more difficult to successfully complete a landing on the remaining runway with the C172M.

Also, notice from Fig. 12 that the total altitude loss for a 195° turnback turn is significantly larger than for either the 210° or 225° turnback maneuvers. This results because of the increased glide distance for the Direct 195° maneuver. In addition, the aircraft lands further down the runway because of the geometry of the 195° maneuver. Furthermore, for the C172 195° Direct maneuver the cushion is larger. Again, the larger cushion may result in reduced ability to successfully complete a landing on the remaining runway.

Aeronca 7AC

Figure 15 shows altitude losses for failure altitudes from 100 to 2000 ft for the Aeronca 7AC. Turnback turns of 195°, 210° and 225° are shown. Again, + is used to indicate 195° and a • for 225°.

Figure 15 is similar to Figs. 11 and 13 yet it is also significantly different. For all three aircraft the altitude loss in the glide parallels the total altitude loss. However, notice that as the ratio of γ_c/γ_g decreases from 1.29 for the E33A, to 0.74 for the C172M and to 0.62 for the 7AC the altitude lost in the glide becomes a larger and larger portion of the total altitue loss. Notice that in Fig. 15 the realignment altitude loss is indistinguishable from the abcissa of the main graph.

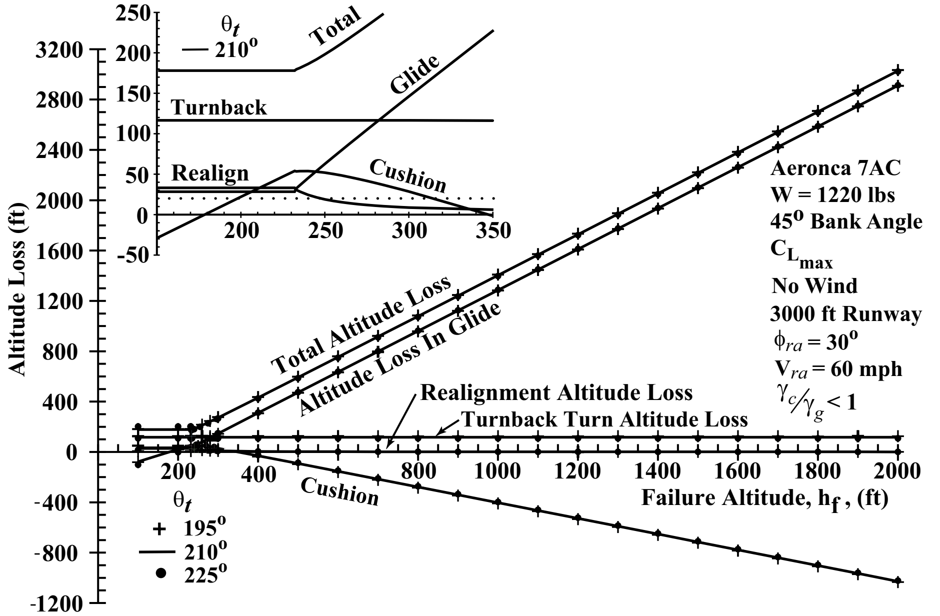

Figure 15. Altitude loss for an Aeronca 7AC for various turnback angles for a 3000 ft runway.

Also recall that for the E33A the cushion increases as the failure altitude increases, while for the C172M the cushion decreases with increasing failure altitude. For the 7AC, Fig. 15 shows that the cushion is initially negative. The cushion then increases with increasing failure altitude and then decreases and again becomes negative. The inset in Fig. 15 illustrates this effect as does Fig. 16

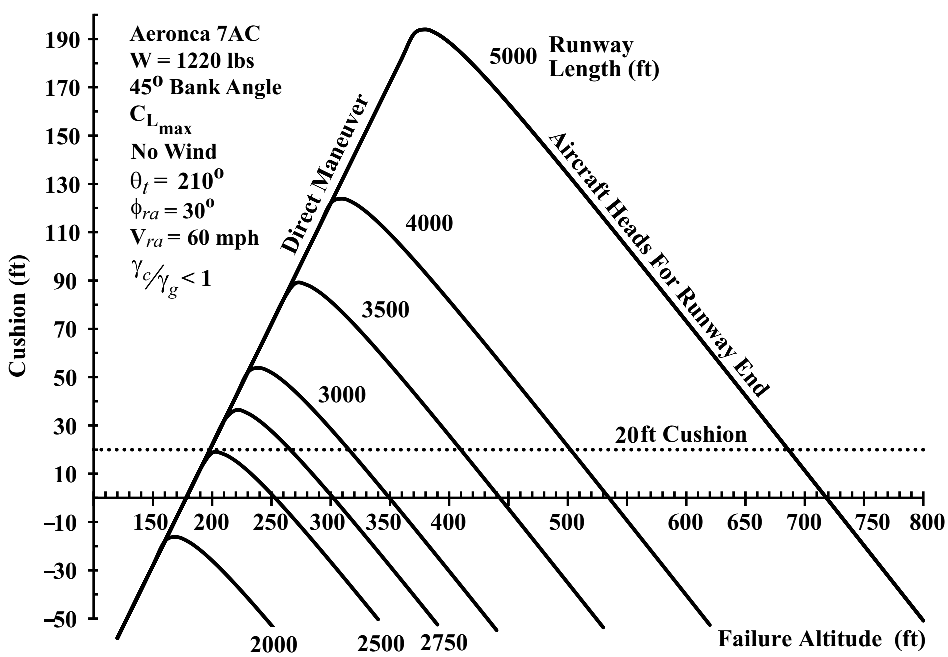

Figure 16. Cushion versus failure altitude for an Aeronca 7AC for various runway lengths.

Clearly, Figs. 15 and 16 indicate that there is a limited range of failure altitudes for which the 7AC can successfully complete the turnback maneuver. Specifically, the 7AC can only complete the turnback maneuver with a 20 ft cushion for failure altitudes between approximately 198 ft and 315 ft. A failure altitude of 198 ft occurs before the departure end of the 3000 ft runway at 2892 ft from the takeoff end. Hence, the Direct turnback maneuver is indicated in Fig. 15. For a failure altitude of 315 ft, failure occurs beyond the departure end of the 3000 ft runway at 4562 ft from the takeoff end. Thus, a classical 210° turnback maneuver is indicated in Fig. 15. In contrast to the E33A and the C172M where the Direct heading occurs for the higher failure altitudes, the Direct heading manuever occurs at lower failure altitudes. For the 7AC, the initial direct heading at lower failure altitudes results because the failure altitude occurs either before the departure end of the runway or is within four radii of the 96.5 ft turnback turn radius beyond the departure end of the runway. Thus, a Direct heading is called for.

For example: for a failure altitude, $h_f = 210$ ft, the distance from the takeoff end of a 3000 ft runway is $x_f = 3063.2$ ft or 63.2 ft from the departure end of the 3000 ft runway. At the end of the turnback turn, the 7AC is at $(x_t, y_t) = (3015,\ 180.2)$ ft. The Direct glide distance to the tangent to the 30° 60 mph realignment turn is just 249 ft. The cushion is 31.9 ft.

For a failure altitude, $h_f = 200$ ft, failure occurs 79.5 ft *before* the departure end of the 3000 ft runway. Here, the cushion is 21.9 ft, 10 ft less than for a failure altitude of 210 ft. The Direct glide distance is still 249 ft, as expected, and illustrated in Fig. 15.

Figure 16 illustrates the effect of runway length on the 'Cushion'. As in Fig. 8, beyond the 'bend' in the curves the aircraft heads for the end of the runway. Prior to that point a Direct maneuver is more optimal. Again, because $\gamma_c/\gamma_g < 1$ the ability of the 7AC to successfully complete a turnback maneuver is limited to the area between the intersections of the curve and the 20 ft cushion line. For example: for a failure altitude of 500 ft off a 5000 ft runway, upon completing a 210° turnback maneuver the aircraft is at an altitude of 134 ft at a point 4983 ft from the takeoff end of the runway. Recalling that the 7AC does not have flaps, the question is: can the aircraft land on the remaining runway? Perhaps, by using an aggressive forward slip. However, neither flight test, nor POH, data for the rate of sink and the forward speed in an aggressive forward slip is known to this author. Consequently, the question is unanswerable with available data.

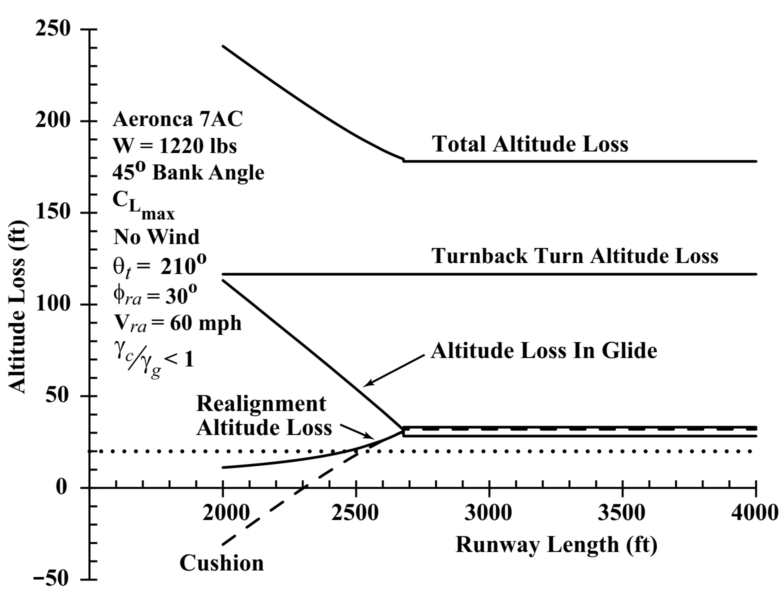

Figure 17. Altitude loss for an Aeronca 7AC for $h_f = 210$ ft and a turnback angle of 210°.

Figure 17 shows the various altitude losses for the 7AC for a failure altitude of 210 ft. Here, in contrast to either the E33A (Fig. 11) or the C172M (Fig. 13), for any altitude that yields a 20 ft cushion, the altitude loss in the turnback turn is the most significant. In addition, for the Direct case the altitude loss in the realignment turn is of similar magnitude to that in the glide.

The 7AC's small turnback radius, low wing loading (W/S = 7.12 lb/ft^2) and low stall speed of 38 mph underlie these characteristics..

General Database Results

Using data from 31 Beech, Cessna, Piper, Mooney and Grumman American pilot operating handbooks along with calculations similar to those above, a number of observations can be made. Keep in mind the diversity of the aircraft in the data set.

For example: the data set contains aircraft as diverse as

light weight low powered fabric covered two seat tail wheel aircraft with a 65 BHP engine and a fixed pitch propeller,

typical all metal four seat fixed gear fixed pitch propeller aircraft,

retractable gear all metal four to six seat aircraft with a constant speed propeller and from 225 to 400 BHP engines,

a composite fixed gear aircraft with a constant speed propeller and a 300 BHP engine.

Gross weights range from 1220 to 3650 lbs. In Fig. 18 the standard deviation in W/BHP is 1.98 and the standard deviation in rate of climb is 294 ft/min. Again, as expected, the maximum rate of climb decreases with increasing power loading W/BHP.

The data set is given in Appendix C.

RATE OF CLIMB

As expected, the maximum rate of climb correlates to the aircraft power loading as shown in Fig. 18. The linear fit shown in Fig. 18 with an $R^2 = 0.84$ is reasonable given the variability of the aircraft in the data set.

ALTITUDE LOSS IN TURNBACK TURN

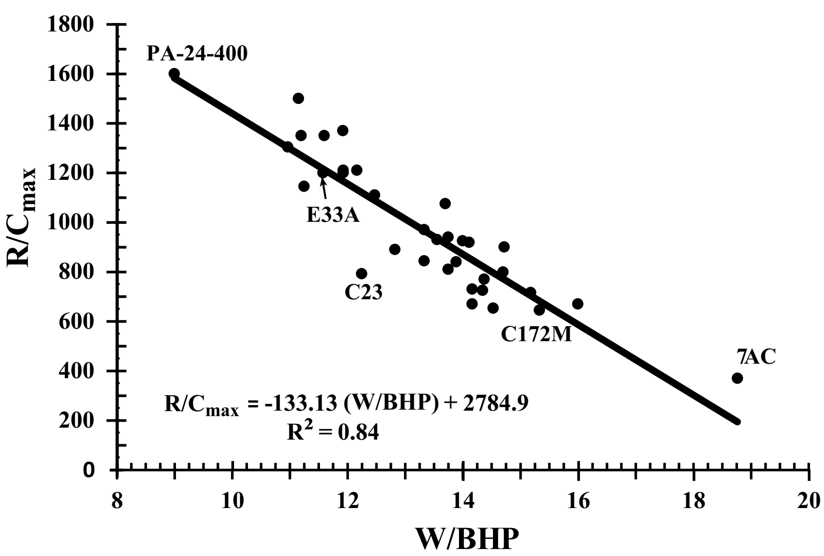

Figure 18. Maximum rate of climb, $R/C_{\max}$ vs power loading W/BHP.

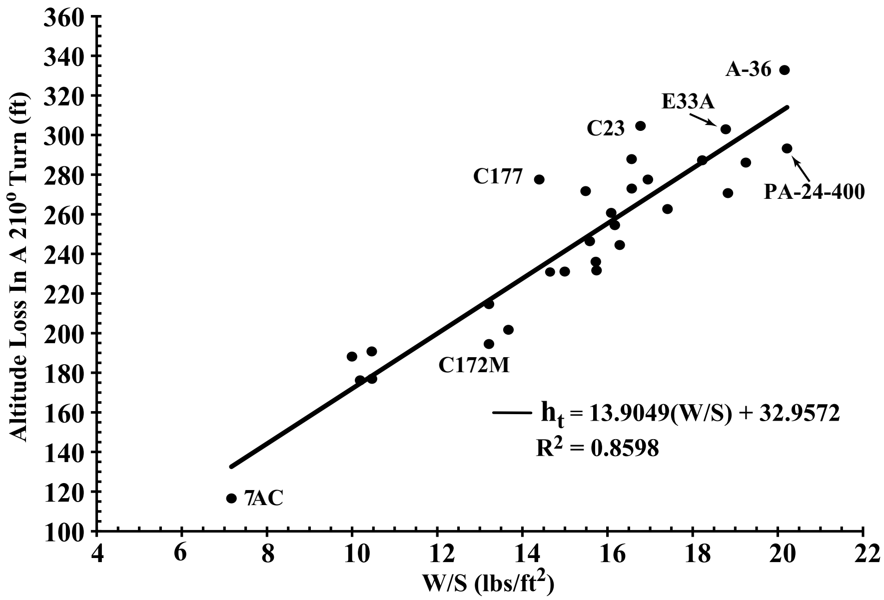

Figure 19. Altitude Loss, h_t, in a 210° turnback turn at C_D/C_L^2 vs wing loading W/S

As shown in Fig. 19 and indicated by Eq. (1), the altitude loss, h_t, in the turnback turn increases with increasing wing load W/S. Again, the $R^2 = 0.86$ value is a result of the diversity of the data set.

The other element of interest in Eq. (1) is C_D/C_L^2. Figure 20 illustrates the variation of C_D/C_L^2 with wing loading, W/S. The solid arrows indicate the data for specific aircraft. The Citabria family of two tandem seat tailwheel aircraft might be considered outliers, as would the C150 and C152 tricycle gear aircraft with two place side-by-side seating. Thus, some deviation from the main body of the data set is not unexpected.

The dashed horizontal line in Fig. 20 indicates the average value of C_D/C_L^2. Clearly the data in Fig. 20 indicate that C_D/C_L^2 is not a significant factor in delineating the altitude loss for the various aircraft in the data set in a gliding turnback turn maneuver.

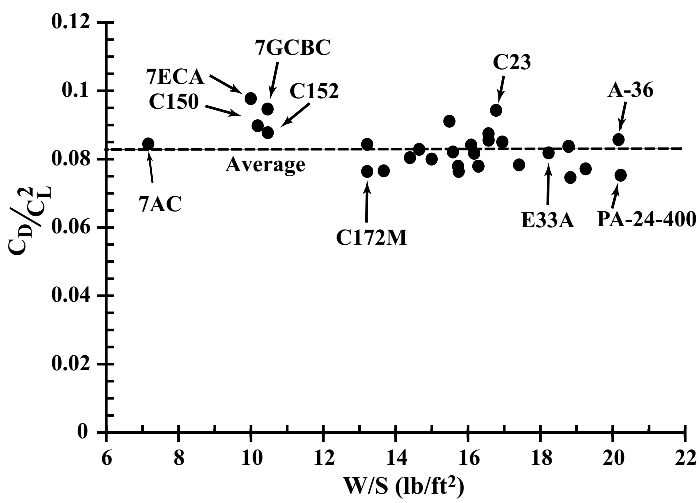

Figure 20. C_D/C_L^2 in a 210° turnback turn vs wing loading W/S.

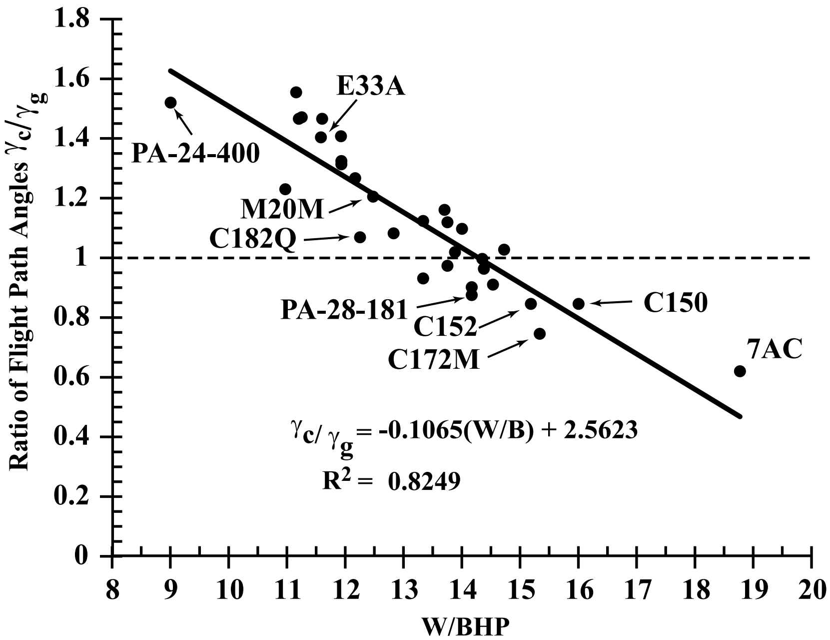

Figure 21. The effect of power loading W/BHP on γ_c/γ_g.

FLIGHT PATH ANGLES

There are two flight path angles of interest. The first is the flight path angle for the initial climb to the failure altitude, γ_c. The second is the flight path angle during the glide toward the runway after completing the turnback turn, γ_g. The magnitude of both γ_c and γ_g increase with increasing rate of climb or increasing rate of sink. More importantly, the speed over the ground, V_{gnd} *decreases*. Hence, the distance over the ground x_{gnd} also decreases if the magnitude of γ_c or γ_g increases.

Increasing aircraft engine horsepower results in increased rate of climb and hence an increase in climb angle, γ_c, provided that the speed for maximum rate of climb does not increase proportionally. This generally occurs when a higher horsepower engine is installed in basically the same airframe, provided that the gross weight increase is not significant. There are numerous examples in the general aviation fleet, especially in the Beech, Cessna and Piper families of single engine aircraft.

As expected, Figure 21 clearly illustrates that the ratio of γ_c/γ_g linearly decreases with power loading, i.e., γ_c/γ_g increases with increasing BHP for a given aircraft weight. Figure 21 labels each of the three examples discussed above, i.e., the E33A, C172M and 7AC. The climb angle to glide angle ratio, γ_c/γ_g for the E33A is above the dashed line at 1.0, while γ_c/γ_g is below the dashed line for the C172M and the 7AC. As the detailed examples above showed, the C172M and the 7AC require relatively longer runways to effect a successful turnback turn and landing. Specifically, the character of the turnback maneuver changes. If $\gamma_c/\gamma_g > 1$, then once the minimum turnback failure altitude is achieved, the aircraft can glide to a successful landing provided that sufficient runway remains. In contrast, if $\gamma_c/\gamma_g < 1$, a successful turnback maneuver is limited to a specific range of failure altitudes.

MAXIMUM LIFT TO DRAG RATIO

Figure 22 shows that, as expected, $L/D_{\max}$ increases with wing loading, W/S. The wing loadings along the line vary from 15 lbs/ft^2 to 20.2 lbs/ft^2. Several aircraft are labelled in Fig. 22. Notice that the two seat training aircraft, e.g., the 7AC and the C150, are clustered at low wing loadings. Clustered at the high end of the wing loading are retractable aircraft, e.g., the Beech E33A/F33A, A36, the Mooney M20M and the Comanche 400. The middle of the range contains a number of fixed gear four place aircraft, e.g., the Cessna 172M, Piper Archer II, and Tiger AA-5B along with a few of the light retractables such as the Piper Arrow and Cessna 177R.

Rules of Thumb

There have been a number of 'rules of thumb' proposed with respect to turnback after engine failure during takeoff in a single engine aircraft. Two of the most cited are the 'One And One Half Rule' and the 'Two-Thirds Rule'. Both are attributable to Barry and Brian Schiff [9-11]. The 'One And One Half Rule' is summarized as:

> at a safe altitude above ground level, establish straight and level flight;
> retard the throttle to idle;
> wait four seconds;
> roll the aircraft into a 45° bank turn;
> continue the turn through 180°;
> arrest the sink to simulate a landing flare;
> note the altitude loss during the maneuver;
> multiple the altitude loss during the 180° turn and 'flare' by 1.5;

the result is the minimum failure altitude, h_f, from which a turnback turn to the takeoff runway should be attempted.

The 'Two-Thirds Rule' is summarized as:

> if you have *not* reached two-thirds of your minimum failure altitude, h_f, as determined above, at the departure end of the takeoff runway the pilot should *not* attempt a turnback to the takeoff runway.

Let's compare these 'rules of thumb' to the current analysis.

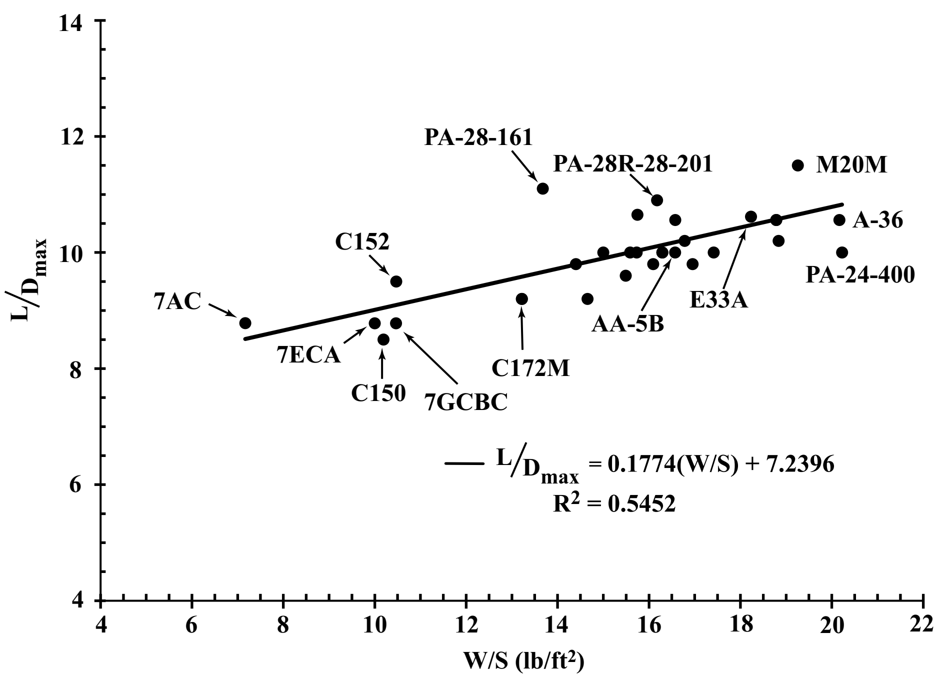

Figure 22. The effect of wing loading W/S on $L/D_{\max}$.

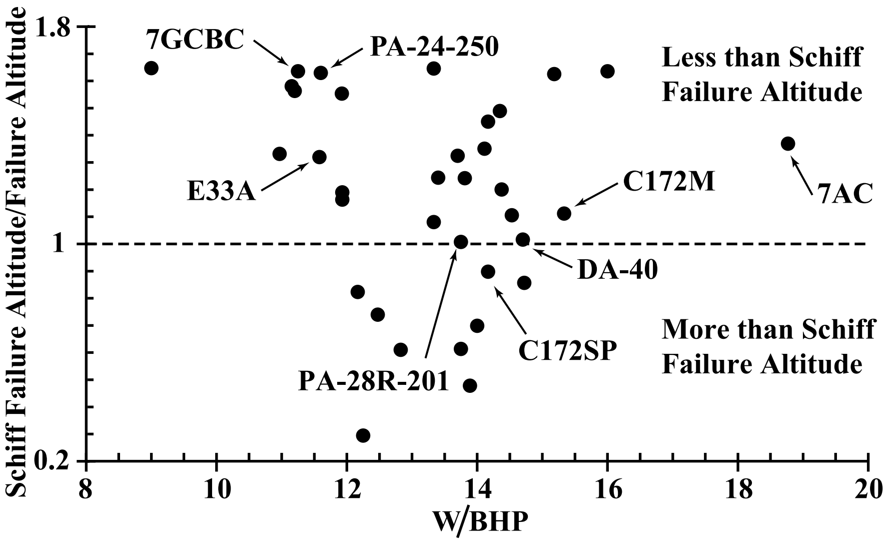

Figure 23. Schiff $1\frac{1}{2} \times 360^\circ$ turn failure altitude 'rule of thumb' compared to the estimated failure altitude, h_f.

One And One Half Rule

In 1974 Barry Schiff [10 & 11] and two professional pilots conducted a series of 180° turnback turn flight tests in four typical fixed gear general aviation aircraft. Specifically, the aircraft were a Piper PA-18A-150 "Super Cub", a Cessna 172L "Skyhawk", a Cessna 185 (with cargo pod), a Cessna 150 "Aerobat" and a Piper PA-28-140 "Cherokee 140". Four bank angles were used, 30°, 45°, 60° and 75°. The aircraft were "heavily" loaded. However, the actual weights for each test are not given. The turnback turns were conducted at the "optimum" speed, i.e., $\mathrm{EAS}_{L/D\max}$ (see [3]), at altitudes between 2000 ft and 4000 ft AGL. The density altitude was not given. Based on the results of the flight tests, Barry Schiff [10] concluded that a 45° bank angle was optimal. Also, based on those results, Barry Schiff recommended that a factor of $1\frac{1}{2}$ be used to estimate the required failure altitude to complete the turnback turn maneuver. No mention was made with respect to required runway length.

Using the C172L/M as an example, Eq. (1) at $\mathrm{EAS}_{L/D\max} = 80$ mph yields an altitude loss for a 180° turn at a 45° bank angle of 265 ft at sea level on a standard day at a gross weight of 2300 lbs. No allowance for arresting the rate of descent was added. Barry Schiff's flight test experiments [9] resulted in an altitude loss of 300 ft, including the 'flare' to arrest the descent — a difference of approximately 13%.

Turning now to the Piper Cherokee 140, Eq. (1) at $\mathrm{EAS}_{L/D\max} = 85$ mph resulted in an altitude loss for a 180° turn of 357 ft at a gross weight of 2150 lbs at sea level on a standard day. Again, there was with no allowance for arresting the rate of descent. Barry Schiff's flight test experiments [9], including arresting the descent, found an altitude loss of 350 ft — a difference of approximately 2%.

Finally, for the Cessna 150 Eq. (1) yields an altitude loss of 223 ft while Barry Schiff's flight test result was 280 ft a difference of 20%.

Figure 23 shows a comparison of the Schiff one and one-half rule of thumb turnback altitude with the turnback altitude, h_f, for the current analysis. Figure 23 clearly shows that, with minor exceptionns, there is no correlation between the Schiff one and one-half rule of thumb and the current engineering analysis. Above the dashed line the Schiff one-and-one half rule of thumb *over-estimates* the required turnback altitude. Below the dashed line the Schiff one-and-one half rule of thumb *under-estimates* the required turnback altitude. The minor exceptions are a 7GCBC, a PA-28R-201 and a DA-40. Notice that the three aircraft are a 'traditional' fixed gear tandem seat tailwheel aircraft (7GCBC), a light retractable side-by-side seat tricycle gear aircraft (PA-28R-201) and a modern fixed gear (with wheel pants) side-by-side tricycle gear aircraft (DA-40).

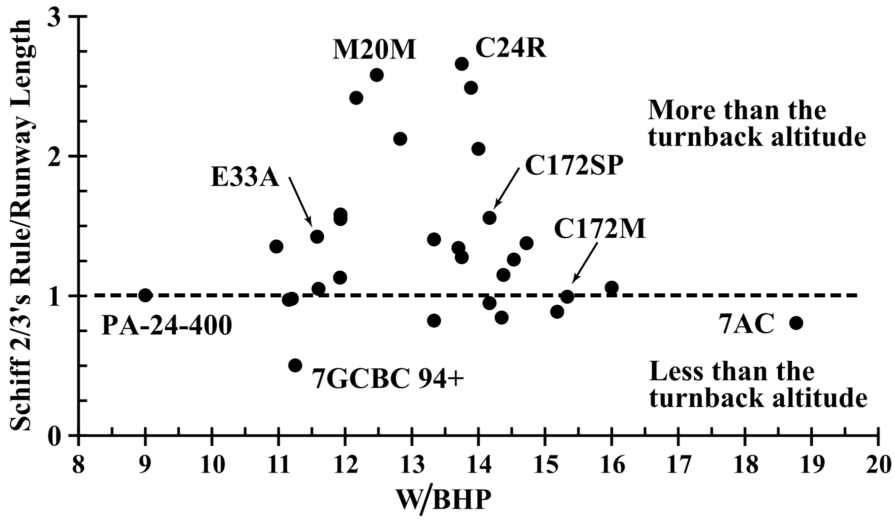

Figure 24. Schiff two thirds 'rule of thumb' compared to the estimated runway length.

The Two-Thirds Rule

Figure 24 illustrates the effectiveness of the Schiff 2/3 rule. Again there is no correlation of the rule of thumb with the estimated runway length determined by the current engineering analysis. Above the dashed line the Schiff 2/3 rule over estimates the failure altitude compared to the current analysis. For aircraft with a $\gamma_c/\gamma_g < 1$ over estimation of the failure altitude, h_f may result in the inability of the aircraft to glide to a successful landing, as shown above. Below the dashed line the Schiff 2/3 rule under estimates the turnback altitude.

WIND EFFECTS

The characterization of the turnback problem as circles over the ground in the no wind case can easily be extended to the wind case. See also the Rogers 1995 paper [2]. Specifically, in the no wind case the third dimension is confined to a single vertical plane. However, in the case with wind, the wind "offsets" the location, over the ground and through the air, to one side and typically closer to the airport if, as is typical, it is a headwind on takeoff. The offset is characterized by the time in the initial climb, during the gliding turnback 'circle' and during the glide, if any, back to the airport/runway. If the wind is variable, either horizontally or vertically, then the flight path may be somewhat 'erratic'. But, if the wind is assumed to be constant with no vertical component, then the flight path is 'smooth', i.e., a smooth curve. The assumption in the Rogers 1995 paper[2] is that the wind is constant with no vertical component.

SPEED AND BANK EFFECTS

As with wind effects, the Rogers 1995 and 2012 papers ([2 and 3]) address the effect of flying the turnback turn above stall speed and at less than a 45° bank angle result in penalties. In both cases, the failure altitude, h_f, and/or the runway length, x_r, required to effect a successful turnback maneuver increases.

Conclusions

The failure altitude and required runway length for a turnback maneuver can be estimated from information found in most pilot operating handbooks.

The turnback maneuver is characterized in terms of the runway length plus four times the turnback turn radius. If the failure altitude, h_f, is within a distance of four times the turnback radius of the departure end of the runway the aircraft should maintain heading Direct to the runway upon completing the turnback turn. Otherwise, the aircraft should head to the departure end of the runway.

The Rogers simplified model [2 and 3] is enhanced to include a realignment turn that aligns the aircraft with the runway centerline at a cushion altitude above the runway.

A turnback turn of 210° is a practical optimum.

Aircraft maximum rate of climb and the climb flight path angle correlates with aircraft power loading.

The ratio of the flight path climb angle to the maximum lift to drag glide angle, γ_c/γ_g, is key to the character of the turnback turn maneuver.

> If $\gamma_c/\gamma_g > 1$, then above a minimum value of the failure altitude, h_f, and minimum runway length the aircraft can execute a successful turnback maneuver.
>
> If $\gamma_c/\gamma_g < 1$, then there may be a limited range of failure altitudes, h_f, and runway lengths within which the aircraft can execute a successful turnback maneuver. In this case, the aircraft typically does not have enough glide range to make the departure end of the runway with an appropriate cushion.

The climb flight path angle to maximum lift to drag glide angle, γ_c/γ_g, correlates well with the aircraft power loading, W/BHP.

The Schiff rule of thumb equating the required failure altitude with one and one-half times the altitude loss observed in a 360° decending turn shows no correlation with the current analysis. [9-11]

The Schiff rule of thumb that "if the aircraft has *not* reached two-thirds of the failure altitude at the departure end of the runway a turnback maneuver should not be attempted" does not show correlation with the current analysis. [9-11]

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Acknowledgments

The author expresses his appreciation to Michael J Bangert (BSAE, MSAE, ATP, CFII) for asking the question "What do I tell my students the failure altitude is?" as well as for his many discussions over the years.

The author also expresses his sincere thanks to John F. Mills (B.Sc.(Eng.), M.Sc.(Eng.), commercial pilot, airplane single engine and multiengine land, instrument airplane, private privileges glider.) who provided many of the original pilot operating handbooks used in this study.

Appendex A

Below is subsection A.11.4 extracted verbatim from FAA Advisory Circular AC 61-83J Appendix A, which provides guidance and recommendations for the preparation and approval of training course outlines.

A.11.4 Return to Field/Engine Failure on Takeoff.

Flight instructors should demonstrate and teach trainees when and how to make a safe 180-degree turnback to the field after an engine failure. Instructors should also train pilots of single-engine airplanes not to make an emergency 180-degree turnback to the field after a failure unless altitude, best glide requirements, and pilot skill allow for a safe return. This emergency procedure training should occur at a safe altitude and should only be taught as a simulated engine-out exercise. A critical part of conducting this training is for the flight instructor to be fully aware of the need for diligence, the need to perform this maneuver properly, and the need to avoid any potential for an accelerated stall in the turn. The flight instructor should demonstrate the proper use of pitch and bank control to reduce load factor and lower the stall speed during the turn. After completing this demonstration, the flight instructor should allow the trainee to practice this procedure under the flight instructors supervision. Flight instructors should also teach the typical altitude loss for the given make and model flown during a 180-degree turn, while also teaching the pilot how to make a safe, coordinated turn with a sufficient bank. These elements should give the pilot the ability to determine quickly whether a turnback will have a successful outcome. During the before-takeoff check, the expected loss of altitude in a turnback, plus a sufficient safety factor, should be briefed and related to the altitude at which this maneuver can be conducted safely. In addition, the effect of existing winds on the preferred direction and the viability of a turnback should be considered as part of the briefing.

Appendix B

Gathered in Table 1 are the data from the POHs of the three example aircraft.

Table 1 Aircraft Data From The Pilot Operating Handbook

Parameter	Symbol	1969 E33A	1974 C172M	1946 7AC
Gross Weight	W	3300 lbs	2300 lbs	1220 lbs
Wing Area	S	181 ft^2	174 ft^2	170.22 ft^2
Wing Span	b	33.5 ft	36 ft	35.15 ft
Stall Velocity (clean)	$\text{EAS}_{\text{stall}}$	72 mph	57 mph	38 mph
V_y Velocity for Max Rate of Climb	$\text{EAS}_{R/C\text{max}}$	112.5 mph	91 mph	60 mph
V_x Velocity for Max Climb Angle	$\text{EAS}_{\gamma\text{max}}$	91 mph	75 mph	50 mph
Velocity L/D_{max}	$\text{EAS}_{L/D\text{max}}$	122 mph	80 mph	60 mph
Max Rate of Climb	R/C_{max}	1200 fpm	645 fpm	370 fpm
L/D_{max} Glide Ratio	L/D_{max}	10.56	9.2	
Oswald Efficiency	e	0.7	0.7	0.6
TO ground run	x_{gnd}	1032.5 ft	865 ft	320 ft(est)
TO distance over 50 ft	x_{50}	1750 ft	1525 ft	780 ft(est)
Speed @ 50 ft	V_{50}	91 mph	68 mph	55 mph(est)
Engine horsepower	SHP	285 hp	150 hp	65 hp
Selected Value				
Turnback Stall Speed Factor	V_{turn}	1	1	1
Turnback Bank Angle	ϕ	45 degs	45 degs	45 degs
Failure Turn Angle	θ_t	210 degs	210 degs	210 degs
Realignment Bank angle	θ_{ra}	30 degs	30 degs	30 degs
Realignment Turn V(Approach Speed)	V_{ra}	100 mph	75 mph	60 mph
Runway length	x_r	3000 ft	4440 ft	3000 ft
Failure Altitude (est)	h_f	682.4 ft	484 ft	198.1 ft

Appendix C

The aircraft the data set used in the analysis consisted of 38 aircraft. Specifically Beech E33A Bonanza (1969), Cessna 172M (1974), Aeronca 7AC (1946), Aeronca 11AC Chief, 1976 Citabria 7ECA, Beech F33A, Beech E33, Beech A33/B33, Beech A-36, Beech V 35B, Beech V K35, Beech C23, Beech C24R, Cirrus SR22, Citabria 7ECA (1975-1977), Citabria 7GCBC (1994 and newer), Cessna 182Q, Cessna 177 (1971), Cessna 177RG (1975), Cessna 172N (1978), Cessna 172SP (1998/2004), Cessna 150 (1975), Cessna 150 (1966), Cessna 152, Cirrus SR22 (2003), Decathlon 8KCAB, Piper Archer II Tapered wing (1998), Piper Arrow, Piper Arrow III (1978), Piper Comanche 250/'58-60, Piper Comanche 250/'62-64, Piper Comanche 260/'66-68, Piper Comanche 260B/'66-68, Piper Comanche 400/'64-65, Piper Comanche 180, Mooney M20M, Mooney M20F (1967), Tiger AA-5B aircraft were included in the database.