Propeller Efficiency
Rule of Thumb

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Theoretically the most efficient propeller is a large diameter, slowly turning single blade propeller. Here, think the Osprey or helicopters. In both cases, large diameter, slowly turning, compared to typical fixed wing aircraft, propellers are used. Generally, single bladed propellers are not used because of dynamic imbalance - think vibration. As a result, the general wisdom is that better propeller efficiency results from decreasing RPM. However, propeller efficiency is not only a function of RPM. It is also a function of propeller diameter and true airspeed. Generally these parameters are combined into a nondimensional parameter called the advance ratio ($J = V/ND$), where V is the true airspeed in feet per second, N is the propeller rotational speed in revolutions per second and D is the propeller diameter in feet.†

Propeller efficiency also depends on the power coefficient, which is a function of, again, N and D and also density as well as the brake horsepower. Specifically, the power coefficient, C_p, is another nondimensional parameter defined by

$$C_p = \frac{\text{BHP}}{\rho\, N^3\, D^5}$$

where BHP is the brake horsepower and ρ (rho) is the local air density. From this, you can see that simply saying lower RPMs give better propeller efficiency is a bit simplistic.

† What is meant by a nondimensional parameter? Well, it is a parameter which, upon substituting the dimensions into the expression for each of the physical parameters, results in all the dimensions cancelling out, e.g.,

$$J = \frac{V}{ND} = V\frac{1}{N}\frac{1}{D} = \frac{\text{ft}}{\text{sec}}\frac{1}{\frac{\text{rev}}{\text{sec}}}\frac{1}{\text{ft}} = \frac{\text{ft}}{\text{sec}}\frac{\text{sec}}{\text{rev}}\frac{1}{\text{ft}} = \frac{\text{ft}}{\text{sec}}\frac{\text{sec}}{\phantom{\text{rev}}}\frac{1}{\text{ft}}$$

Because revolutions (rev) is not a physical dimension, the denominator in the second term is replaced with a blank. Finally, we have

$$J = \frac{V}{ND} = \frac{\text{ft}}{\text{sec}}\frac{\text{sec}}{\text{ft}} = \frac{\cancel{\text{ft}}}{\cancel{\text{sec}}}\frac{\cancel{\text{sec}}}{\cancel{\text{ft}}}$$

and each of the physical dimensions cancels out, i.e., J is dimensionless.

Furthermore, as with any aircraft, the designer has a design goal in mind. For the Bonanza, the design goal was high speed cruise coupled with all around good handling and performance. The design goal influences propeller design and selection.

Propellers

Typically propellers are divided into three main categories: fixed pitch, adjustable (controllable) pitch, both ground and in flight adjustable, and constant speed (RPM). Because of wartime experience, Beech originally chose a controllable pitch propeller for the Bonanza. Maximum propeller diameter is principally influenced by ground clearance and tip speed (Mach number). Bonanza propellers started at 88 inches in diameter and, except for takeoff, a maximum RPM of 2050. As maximum engine RPM increased, diameter decreased, because of tip Mach number, to 80 inches at 2700 RPM for a constant speed propeller.

The basic design philosophy for a constant speed propeller is, for any selected engine power, or torque, to change the pitch (angle) of the propeller blades to absorb the selected engine power, provided there is enough torque to turn the propeller at the selected RPM. Increasing the blade pitch increases the blade drag, while decreasing the blade pitch decreases the blade drag. Hence, a larger (coarser) blade angle, for a given RPM, will absorb more power and require more torque to turn it at the requested RPM. Similarly a smaller (finer) blade angle, for a given RPM, will absorb less power and require less torque to turn it at the requested RPM.

Propeller blades are twisted from root to tip. The amount by which the blades are twisted, along with the variation in chord, airfoil section and sweepback of the blade leading edge, are design decisions. Those design decisions are significantly influenced by the design goal. Even with a controllable pitch or constant speed propeller, optimum design throughout the flight regime is not achievable. The design goal for the Bonanza is high speed cruise. Hence, propeller design and selection is optimum or near optimum for high speed flight. Thus, performance for takeoff and climb is suboptimal.

A Simplified Rule of Thumb

From the propeller maps for both the common two and three blade McCauley propellers fitted to the later Bonanzas, a *generic* propeller efficiency curve, η, (eta) as a function of the advance ratio, J, can be estimated as shown in Figure 1. For full power and 2700 RPM at sea level and for 65% power at 6000 ft at 2300 RPM, the maximum propeller efficiency occurs for between $J = 0.95\pm$ and $J = 1.05\pm$ as shown by the gray band in Figure 1.

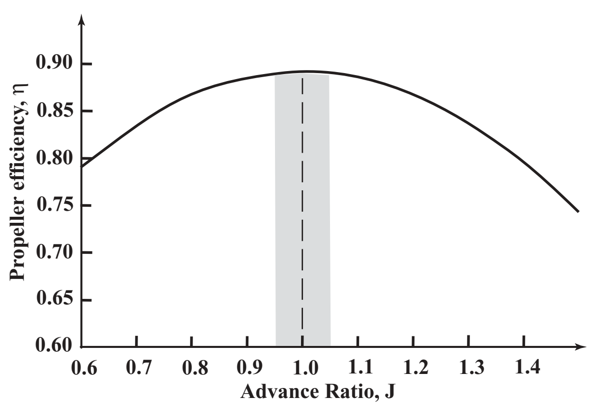

Figure 1. Generic propeller efficiency vs advance ratio.

Outside of these values the propeller efficiency decreases. From this, we can see that, for a fixed diameter propeller, it is the ratio of the true airspeed to the propeller RPM that is important in achieving maximum efficiency.

For this generic propeller efficiency curve the ratio of RPM/TAS should be maintained at approximately 15 to give an advance ratio of $J = 1.0\pm$, i.e.,

$$J = \frac{V}{ND} = \frac{1.69\,\mathrm{KTAS}}{(\mathrm{RPM}/60)(\mathrm{D}/12)} = \frac{(60)\,(1.69)\,(12)\,\mathrm{KTAS}}{(\mathrm{RPM})\,(\mathrm{D})} = 1.0$$

where the 1.69 converts KTAS (knots true airspeed) to ft/sec TAS, the 60 converts RPM to RPS (revolutions per second) and the 12 converts inches to feet. For a propeller diameter of 80 inches, after rearranging and inverting this equation, we have

$$\frac{RPM}{KTAS} = 15.2$$

as a rule of thumb to maintain maximum propeller efficiency. However, this is a rule of thumb so let's use 15.0 for A Simplified Rule Of Thumb (ASROT).† It is easier to remember and close enough.

Fine Tuning the Rule of Thumb

The propeller efficiency curve shown in Figure 1 is a composite of both the typical Bonanza two and three blade propellers at two different conditions. Is there a way to fine tune the rule of thumb for a specific aircraft and propeller? Because the Bonanza design goal was high speed cruise, it is reasonable to assume that the factory propeller is optimized for that condition.

As a practical matter, for a normally aspirated engine the cruise true airspeed increases with altitude until the 'critical' altitude for a given power setting is reached. Let's call the critical altitude the 'knee' in the curve. Above the knee the engine can no longer produce the requested percentage of power. High speed cruise conditions for various altitudes and power settings are represented by the altitude vs cruise airspeed graph (see Figure 2) in the performance section of the Pilot Operating Handbook (POH). As examples to test the ASROT let's use the knee in the altitude vs cruise true airspeed graph from the POH.

Table 1 Cruise Airspeed Efficiencies

Altitude	%BHP	BHP	RPM	KTAS	J	C_p	η	Symbol	ASROT
6000 ft	75	213.8	2500	173	1.059	0.0621	0.902	Red dot	14.5
7500 ft	65	185.3	2300	164	1.085	0.0724	0.905	Blue dot	14.0
8700 ft	55	156.8	2100	152	1.101	0.0722	0.905	Green dot	13.8
14000 ft	45	128.3	2100	141	1.021	0.0807	0.896	Black dot	14.9
11000 ft	45	128.3	2100	140	1.010	0.0722	0.900	Red circle	15.0

Weight = 3100 lbs, standard day

Table 1 illustrates the results for the power settings, altitudes and airspeeds corresponding to the knee in the cruise true airspeeds vs altitude curves represented by the dashed line in Figure 2 obtained using the bare propeller efficiency map‡ for the C76 McCauley 3-blade 80 inch propeller, as shown in Figure 3. Table 1 shows that the advance ratio, J, lies on the plus side of the 0.95 to 1.05 maximum efficiency curve at the higher true airspeeds and

† Pronounced AS ROT

‡ Bare propeller efficiencies do not consider blockage effects of the nose or nacelles. For a properly designed propeller blockage effects can *decrease* propeller efficiencies by 1-3%.

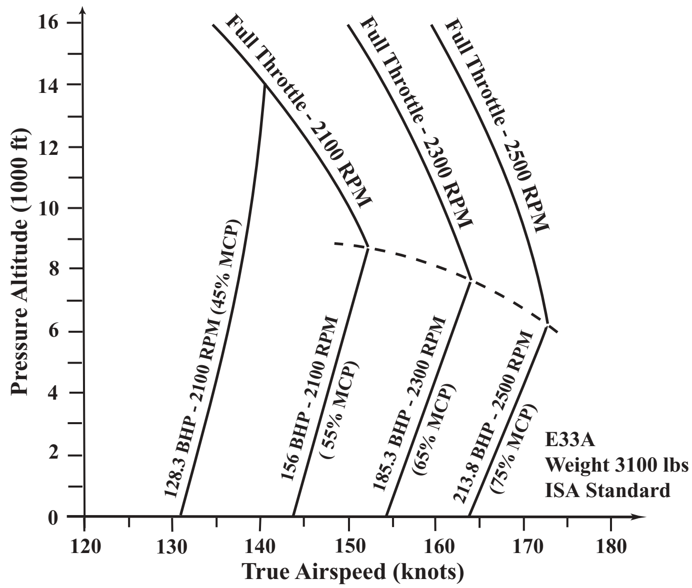

Figure 2. Cruise airspeeds.

RPMs. This implies that an ASROT factor of 15 over-estimates the required RPM. The last column in Table 1 shows the ASROT factor required to obtain the stated RPM at the stated KTAS, i.e., the RPM in column 4 divided by the KTAS in column 5. The result shows that the ASROT factor for this propeller should be adjusted downward. A reasonable adjusted ASROT factor of 14.5 is suggested.

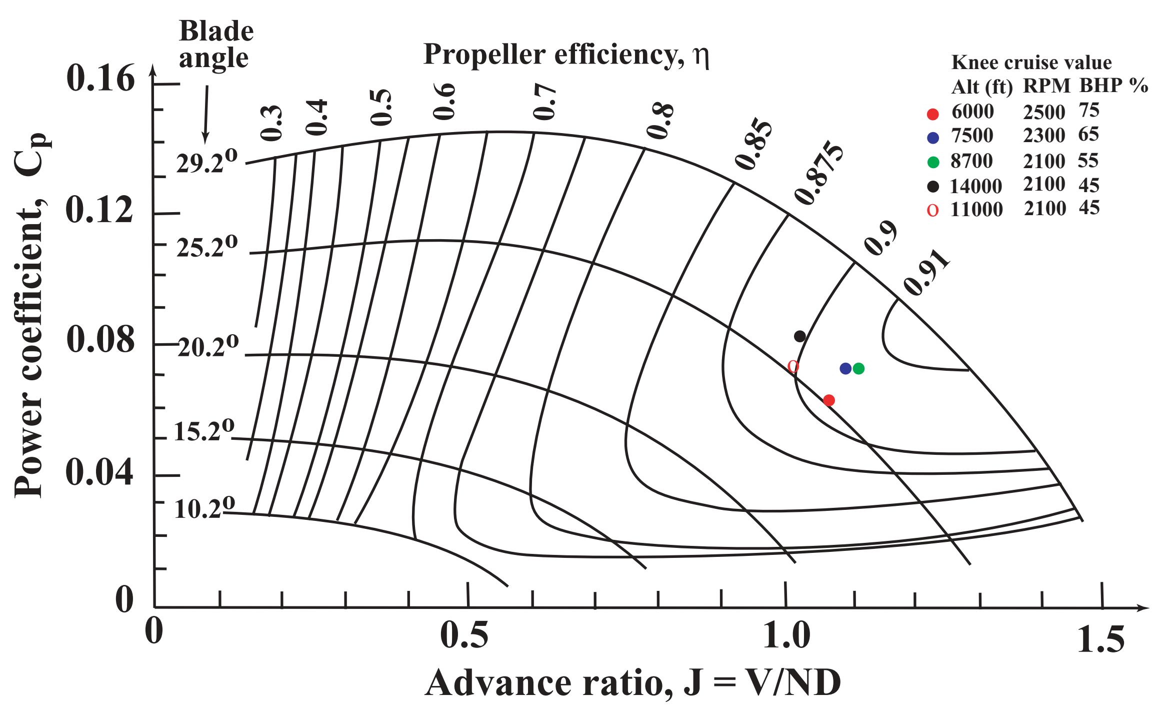

Figure 3. McCauley C76 propeller map.

Table 1 also shows that typically for higher KTAS higher RPM is required to maintain maximum propeller efficiency, while for lower KTAS lower RPM is required to maintain maximum propeller efficiency. This result confirms the current wisdom that lower RPM for lower true airspeeds increases propeller efficiency.

Turning now to the C76 propeller map shown in Figure 3, notice the clustering of all of the high speed cruise data from 6000 to 14,000 ft. In fact, Figure 3 and Table 1 illustrate that the difference in propeller efficiency is at most a little over $1/2$%. This suggests that using the knee values from Figure 2 is a reasonable way to fine tune the ASROT value for a particular propeller. It also indicates that being a little off in either RPM or J is not serious.

Finally, be aware that determining the specific RPM and manifold pressure that will give near optimal propeller efficiency for any given flight condition is an iterative process.

Hang on to Figure 3, we'll be coming back to it when we look at take-off, climb and turbonormalized operations.