Wind and Range

David F. Rogers

In a previous article (Range) we discussed how to determine the optimum velocity to achieve maximum range. That discussion was limited to the no wind case. In the no wind case, the velocity for maximum range is

$$\mathrm{EAS}_{\mathrm{L/D_{max}}} = \sqrt{\sigma}\,\mathrm{TAS}_{\mathrm{L/D_{max}}} = \Big(\frac{2}{\rho_{\mathrm{SL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}\Big)^{1/2}$$

where

b is the wing span
e is the airplane efficiency factor
f is the equivalent flat plate area
W is the weight
ρ_{SL} is the density at sea level on a standard day
σ is the density ratio
EAS is the equivalent airspeed
TAS is the true airspeed

We seldom fly in no wind—for me it is always a headwind! Thus, we are interested in the effect of wind on the optimum velocity for maximum range. The basic concept is that in a headwind you fly faster to decrease the time that the headwind has an effect, while in a tailwind you fly slower to increase the time that the tailwind has an effect. The real question is, how much slower or faster need you fly?

The classical technique uses that oh so useful power required curve. Recall from previous articles that for no wind the velocity for maximum range is found by drawing a line through the origin tangent to (just touching) the power required curve, as shown by the solid line in Figure 1.

Range is the distance over the ground and the wind affects ground speed and not airspeed. A headwind decreases the ground speed, while a tailwind increases the ground speed for a given airspeed. On the power required versus velocity plot, the decrease in ground speed is represented by moving the starting point of the tangent line to the right along the horizontal axis an amount equal to the wind speed. This is shown by the dotted line in Figure 1. For a tailwind, the starting point of the tangent line is moved to the left an amount equal to the wind speed, as shown by the dashed line in Figure 1. Notice that, as expected, the velocity for maximum range in a headwind is increased, while that for a tailwind is decreased. However, also notice that, because the curve is very 'flat' in this region, it is difficult to determine by exactly how much the velocity changes. Besides, even I do not draw power required versus velocity curves as I am happily flying along! So, we need some rules-of-thumb to guide us. But we need to have a solid basis for those rules-of-thumb, not just hangar talk.

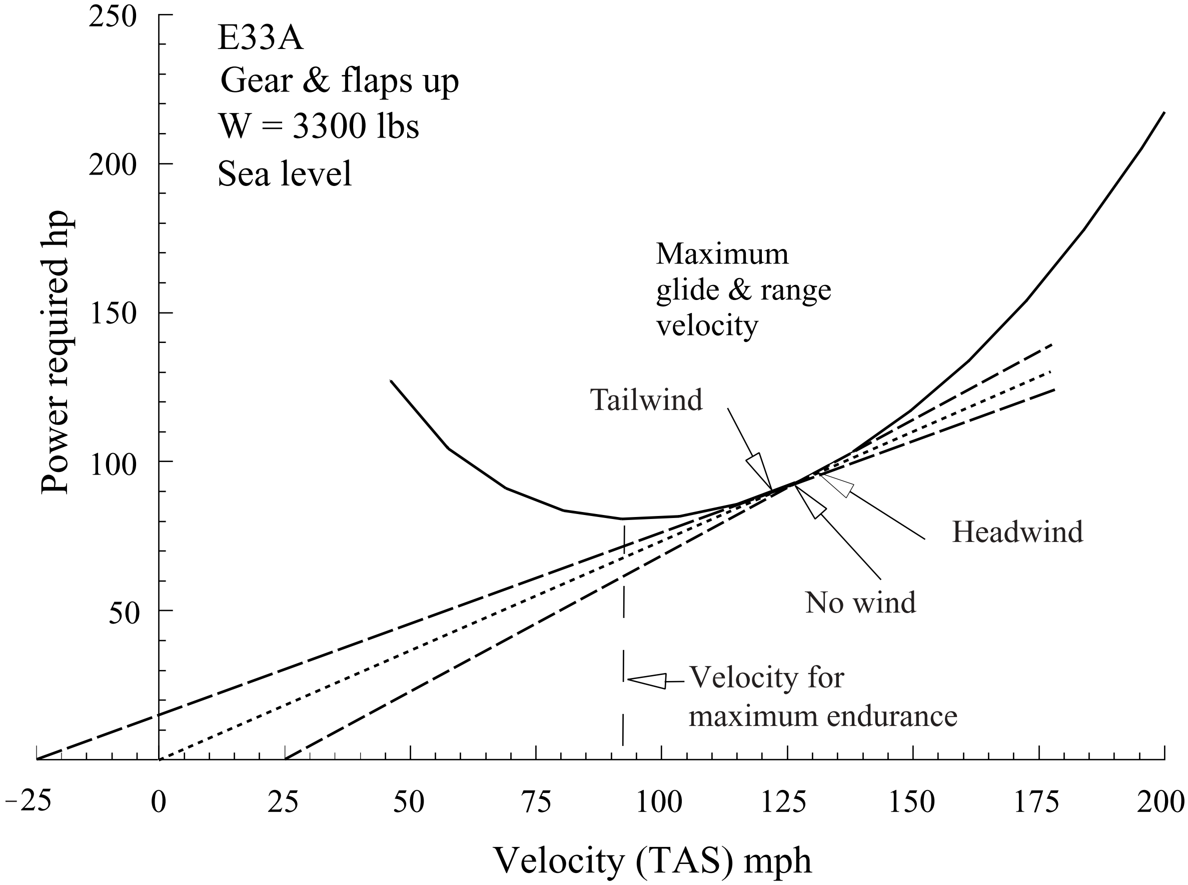

Figure 1. Wind effects from the power required curve.

A number of mathematical solutions are available. The resulting equations are rather complex—no I am not going to show them here. The results of one of the solutions, due to Professor Philip Bridges of the Mississippi State University, is shown in Figure 2. In Figure 2, V/V_o is the ratio of the velocity to fly in the wind to the velocity for maximum range with no wind; and w/V_o, called the wind fraction, is the ratio of the wind velocity to the velocity for maximum range with no wind, V_o. V_o is the best glide velocity given in the POH. Remember that the best glide velocity given in the POH is for maximum gross weight. Thus, it should be corrected for weight effects as discussed in a previous article (Weight Effects, Part I, pp. 18–21, Jan/Feb '97). w/V_o is negative for a headwind and positive for a tailwind. The solution is accurate when w/V_o is between -0.5 and $+0.5$, i.e., when either the tailwind or headwind is up to half the no wind velocity for maximum range. Notice that the solution says that the effect of a headwind is greater than that of a tailwind.

When flying into a headwind, a typical rule-of-thumb used by sailplane pilots is to add half of the headwind velocity to the no wind best glide velocity. We'll call this the half-rule-of-thumb. The dashed line in Figure 3 shows the half-rule-of-thumb compared to the analytical solution due to Bridges represented by the solid line. Looking at Figure 3 shows that the half-rule-of-thumb yields too large a velocity increase, especially at the larger headwind fractions, i.e., higher headwinds.

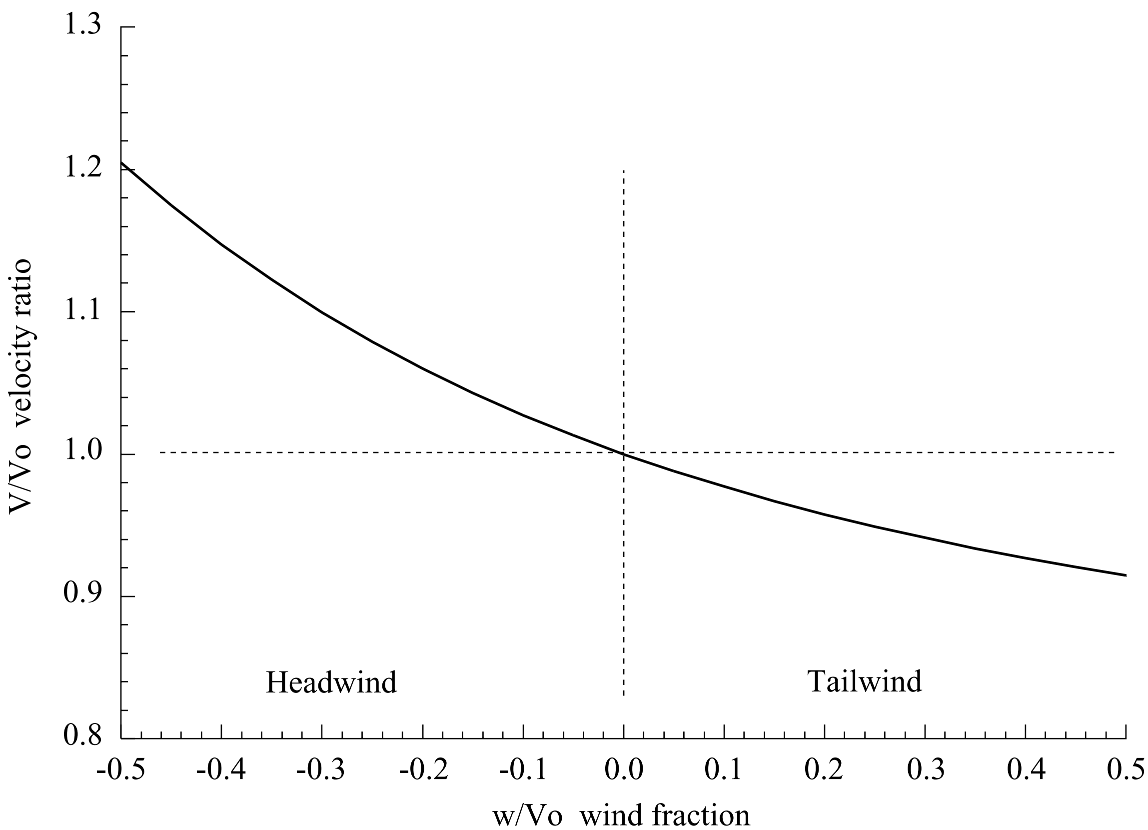

Figure 2. Mathematical solution for the maximum range velocity.

The half-rule-of-thumb is typically not used for tailwinds. However, if we used the half-rule-of-thumb to determine the decrease in best range velocity for a tailwind, Figure 3 shows that the estimate is even worse.

For wind fractions up to about a quarter of the no wind best range velocity, a better approximation to the analytical results, for both headwinds and tailwinds, is to add or subtract a quarter of the headwind or tailwind velocity. This is called the quarter-rule-of-thumb represented by the dotted line in Figure 3.

For example, for a model E33A Bonanza the best glide velocity is 122 mph. When flying into a 30 mph headwind, using the analytical solution given in Figure 2 we have $V/V_o = 1.08$ for $w/V_o = 30/122 = 0.246$. Hence, the velocity for best range is $1.08 \times 122 = 132\,$mph. The half-rule-of-thumb says you should add 15 mph and fly at 137 mph $(122 + 30/2)$ to obtain the best range. The quarter-rule-of-thumb says you should add 7.5 mph and fly at 129.5 $(122 + 30/4)$ mph which is closer to the analytical result.

As a further example, consider the same 30 mph wind but now a tailwind—don't we all wish. From Figure 2 for $w/V_o = 0.246$ we have $V/V_o = 0.95$ and the best range velocity is $0.95 \times 122 = 116\,$mph. The half-rule-of-thumb to subtract 15 mph and fly at 107 mph $(122 - 30/2)$, while the quarter-rule-of-thumb says to subtract 7.5 mph and fly at 114.5 mph $(122 - 30/4)$. Again, the quarter-rule-of-thumb is closer to the analytical result.

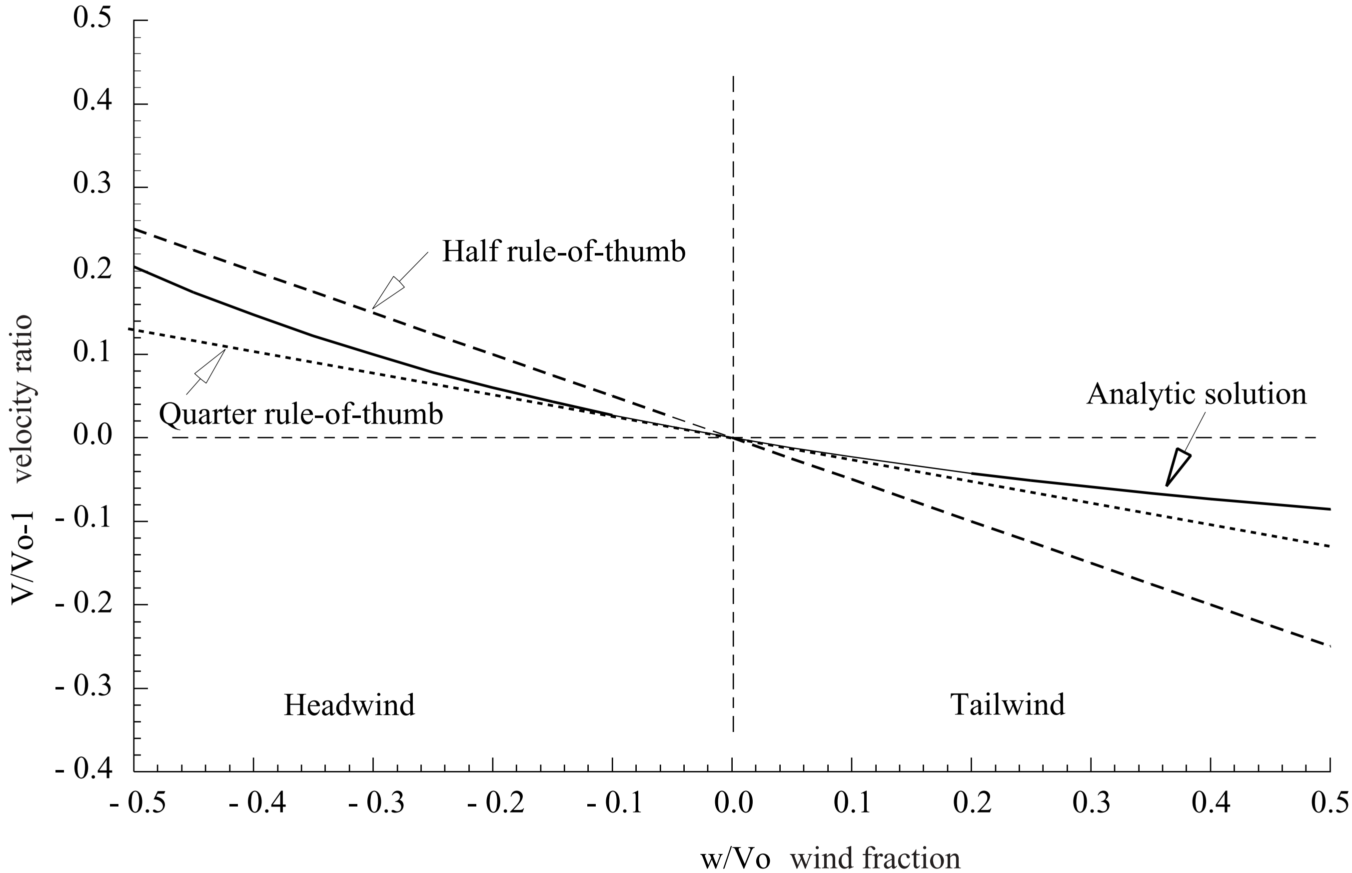

Figure 3. Comparison of rules-of-thumb tothe mathematical solution.

However, Figure 3 also shows that, for headwind fractions greater than a quarter, the quarter-rule-of-thumb gives too low a value. Hence, you need to increase the best range speed somewhat. What this really says is that the best range speed in the wind is a nonlinear problem, i.e., it cannot be represented by a simple straight line rule-of-thumb applicable for all wind fractions.

One final point. In a significant tailwind the minimum velocity at which you should fly is the velocity for minimum power required, which is also known as the velocity for maximum endurance. This velocity represents the maximum time that an aircraft can remain aloft for a given amount of fuel. The velocity for maximum endurance is 76% of the no wind maximum range velocity. For example, for a model E33A Bonanza the maximum endurance velocity is $0.76 \times 122 = 93\,\text{mph}$. Returning to Figure 1, notice that if you fly at a velocity less than the velocity for maximum endurance that the power required to maintain level flight increases. Thus, your fuel burn also increases and your range decreases.

