Turn Performance Sustained Level Turns

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In the turning article we looked at steady level turns. There we found that the radius of the turn depends only on the velocity (TAS), i.e., $R = V^2/g \tan\phi$. However, we also remarked that you do have to maintain that steady level turn. We all know that the stall velocity increases as the bank angle increases. Hence, the stall velocity in the turn provides an obvious lower limit. In addition, the power required to maintain the steady level turn also increases as the bank angle increases; thus, the power available also limits turn performance. Here we look at these effects.

From the turning article recall that, in a turn, the sum of the forces in the vertical direction is

$$(L + T\sin\alpha_T)\cos\phi = W \tag{1}$$

Now, if for simplicity we neglect the term $T\sin\alpha_T$ as small, we have $L\cos\phi = W$. Rearranging yields

$$L = \frac{W}{\cos\phi} = nW \tag{2}$$

where n, called the load factor, is $1/\cos\phi$. Notice from Table 1 that for a bank angle of 60° the load factor is doubled while at small bank angles the increase is quite small.

What this result tells us is that, because of the bank angle, the lift that the aircraft must generate in order to sustain level flight is increased by the load factor. If the required lift is increased, then the induced drag and the effective induced power required to sustain level flight are also increased. Alternatively we can consider that the effective weight of the aircraft is increased by the load factor.

Recalling the power-required curve for steady level flight

$$P_r = \underbrace{\text{Constant}\,\sigma f}_{\text{parasite}} + \underbrace{\frac{\text{Konstant}}{\sigma}\left(\frac{W}{b}\right)^2}_{\text{effective induced}}$$

where σ represents the ratio of density at any altitude to the density at sea level, f is a measure of the parasite drag, W is weight, b is the wing span and Constant and Konstant are constants. Now in a steady level *turn* the effective weight is increased

Table 1. Cosine functions.

ϕ	$1/\sqrt{\cos\phi}$	$1/\cos\phi$	$1/(\cos\phi)^2$
0	1.000	1.000	1.000
5	1.002	1.004	1.008
10	1.008	1.015	1.031
15	1.017	1.035	1.072
20	1.032	1.064	1.132
25	1.050	1.103	1.217
30	1.075	1.155	1.333
35	1.105	1.221	1.490
40	1.143	1.305	1.704
45	1.189	1.414	2.000
50	1.247	1.556	2.420
55	1.320	1.743	3.040
60	1.414	2.000	4.000

by the load factor. Thus, the effective induced power required is increase by the load factor *squared.* The power required in a steady level turn is now

$$P_{r_{\text{turn}}} = \underbrace{\text{Constant}\,\sigma f}_{\text{parasite}} + \underbrace{\frac{\text{Konstant}}{\sigma}\left(\frac{W}{b}\right)^2 \frac{1}{(\cos\phi)^2}}_{\text{effective induced}}$$

Table 1 shows that at larger bank angles the increase in effective induced power required is quite significant. For example, at a 60° bank angle the effective induced power required is four times that in steady level flight at zero degrees bank angle. For a given power available, the increase in effective induced power required limits the maximum velocity that can be sustained in a banked level turn, as shown by the intersection of the power available and power required curves in Figure 1.

In most cases the stall velocity represents the minimum velocity in a sustained steady level turn. Recall that the stall velocity is given by

$$V_{\text{stall}_\phi} = \sqrt{\frac{2W/S}{\sigma\rho_{\text{SL}}C_{L_{\max}}}}\sqrt{\frac{1}{\cos\phi}} = \frac{V_{\text{stall}}}{\sqrt{\cos\phi}}$$

where V_{stall_ϕ} is the stall velocity in a level turn at the bank angle ϕ, V_{stall} is the stall velocity in steady level flight with $\phi = 0$ and $C_{L_{\max}}$ represents the aircraft maximum lift coefficient. Thus, in a sustained level banked turn the stall velocity increases as one over the square root of the cosine of the bank angle. Both the curve of stall velocity and that of stall velocity plus ten miles per hour are shown in Figure 1. In the light gray shaded area the aircraft can sustain a steady level turn at a power available at or below the maximum power available. If, for a particular velocity and bank angle, the power available exceeds the power required, then the aircraft climbs. Notice, as shown by the dark gray shaded area in Figure 1, that even in a 60° bank a positive rate-of-climb exists for a small range of velocities. However, notice that in a

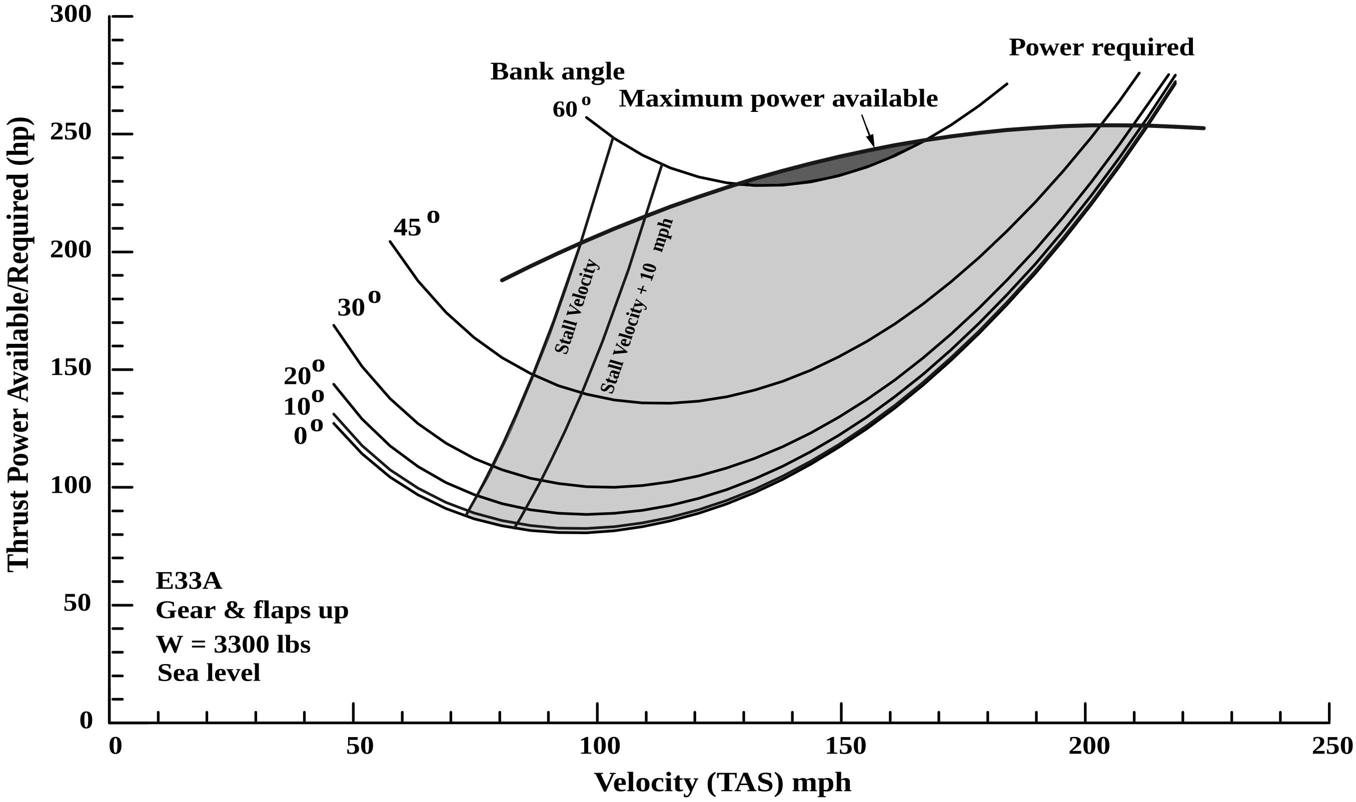

Figure 1. Power required and available in a steady level coordinated turn.

60° level bank at the stall velocity the aircraft will *not* climb. In fact, it will descend, because the power required exceeds the power available. Here, the lower velocity limit for a level turn is determined by the power available.

Figure 2 represents the gray area of Figure 1. The outer boundaries of the curve are the bank angle and velocity for a level turn, while the interior of the curve represents velocity bank angle pairs where the aircraft has excess power available for climb. Figure 2 shows that the maximum bank angle in a level turn is a bit less than

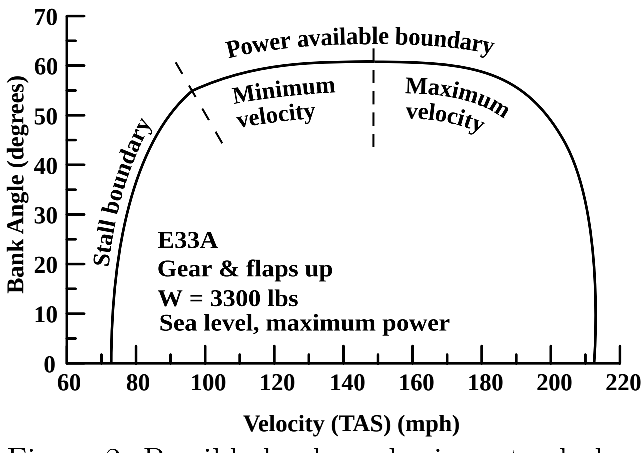

Figure 2. Possible bank angles in a steady level turn.

61° at 149 mph. The 'knee' in the curve where the power available begins to limit the minimum velocity occurs for $\phi = 55°$ and 96.4 mph. Increasing altitude collapses the curve inward.

We'll look at altitude effects on the available bank angles and velocities for a level turn in the Turning Performance—Altitude Effects article.