Two-Chart Normally Aspirated Performance - Part 1

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Introduction

There are many facets to aircraft performance. Here, steady state performance is considered. Steady state performance implies that the aircraft is not accelerating. Steady state aircraft performance typically encompasses steady level flight, steady climbing flight and steady descending flight, whether the aircraft heading is not changing or the aircraft is in a turn at a constant bank angle and the heading is changing. The material is also applicable to what engineers call quasi-steady state flight. Quasi-steady state flight implies that the aircraft is changing from one steady

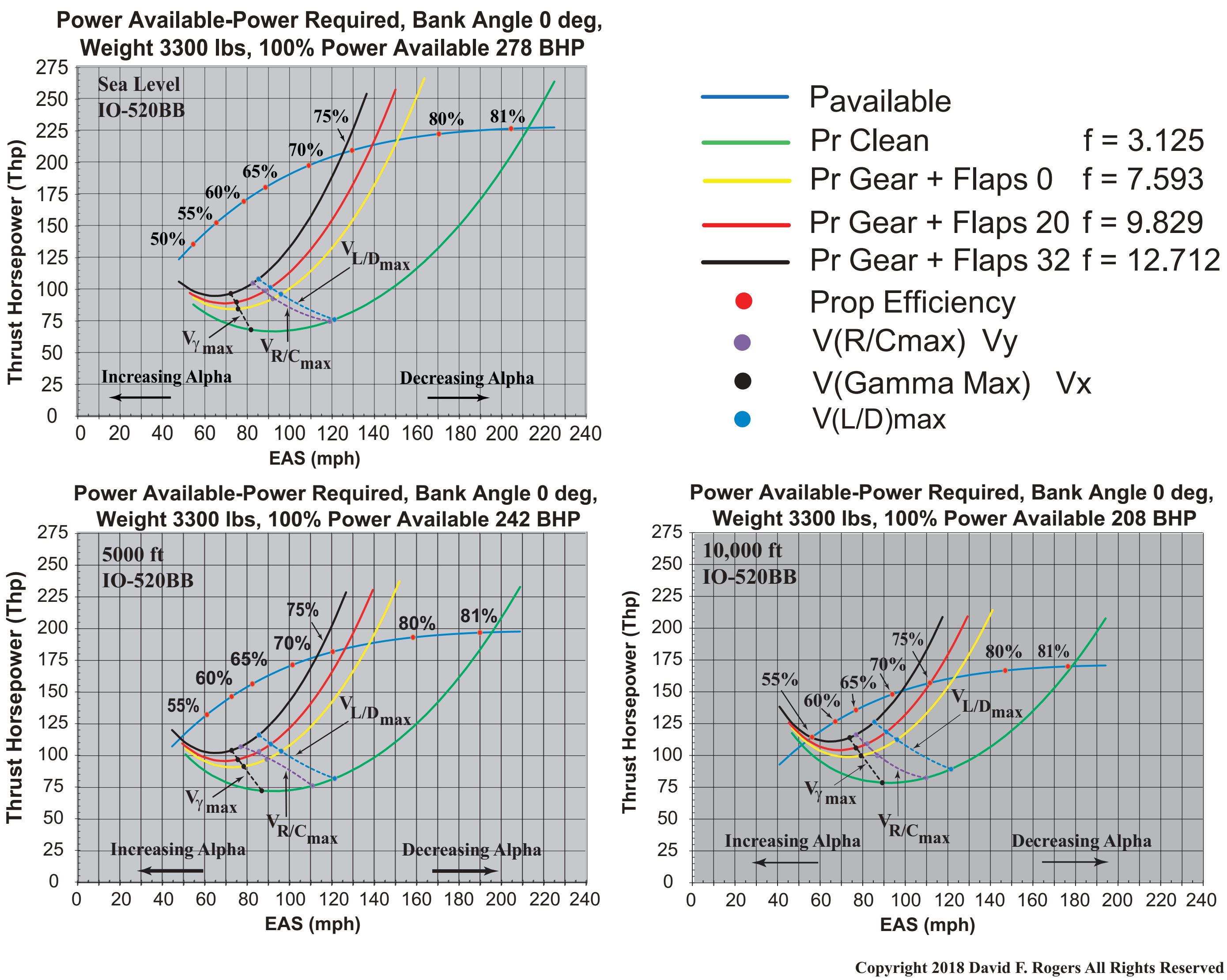

Figure 1. Thrust horsepower available and thrust horsepower required for a normally aspirated aircraft in steady flight without a heading change.

state condition to another slowly. For example: The aircraft may be slowly changing from one steady level flight condition to another steady level flight condition at a rate of perhaps one knot per second. Another example is an aircraft approaching a stall at the FAA (Federal Aviation Administration) standard deceleration rate of one knot per second. However, when the aircraft stalls, the actual stall and recovery is not amenable to quasi-steady state analysis.

Chart 1: Steady Flight Without A Heading Change

The aircraft modeled here is an E33A Bonanza high performance single engine retractable aircraft equipped with flaps having a maximum deflection angle of 32°.

Figure 1 shows Chart 1 with three graphs for thrust horsepower required curves for steady level flight on a standard day at sea level, 5000 ft and 10,000 ft for the clean (gear and flaps up), gear down, gear down and flaps deflected 20° and gear down and flaps deflected 32°. These are the green, yellow, red and black J shaped curves respectively shown in Fig. 1.

Let's continue to take this chart apart piece by piece. Why? Because this chart represents what you really need to understand about aircraft performance.

The arrows at the bottoms of the graphs indicate that as Equivalent Airspeed (EAS) increases, the angle of attack α (Alpha) decreases and as EAS decreases the angle of attack increases.

The horizontal axis in Fig. 1 is EAS which, in this case, is the same as Calibrated Airspeed (CAS) for all practical purposes. This is a good place to point out that equivalent airspeed is used throughout this discussion. As a reminder, true airspeed, TAS, except at sea level on a standard day, is greater than equivalent airspeed. Specifically

$$TAS = \frac{EAS}{\sqrt{\sigma}\rho_{\text{SSL}}} \tag{1}$$

where σ (Sigma) is the ratio of density at altitude on a standard day to that at sea level on a standard day, ρ_{SSL}. At sea level $\sigma = 1.0$. At any other altitude on a standard day $\sigma < 1$, i.e., smaller than at sea level. Hence, TAS is higher than EAS at any altitude above sea level.

The blue curved line that intersects the J shaped thrust power required curves represents the thrust horsepower available. Thrust horsepower available (THP_{avl}) is the brake horsepower (BHP) times the propeller efficiency, η (eta), i.e.

$$THP_{\text{avl}} = \eta BHP \tag{2}$$

The intersections of the THP_{avl} and the THP_{req} curves represent the minimum and maximum steady level flight speeds for a given thrust horsepower available and the thrust horsepower required for a given configuration. Between these speeds the aircraft has excess energy available for maneuvers, i.e., a steady climb.

The values of f in the legend represent the parasite drag contribution for the various configurations.[*]

Notice that the thust power required curves move up and to the left as the parasite drag increases with changes in aircraft configuration. Also, notice that the curves move up and to the left with increasing altitude.

The % numbers along the THP_{avl} curve represent the efficiency of the propeller at the points along the curve indicated by the red dots.[†] Propeller efficiency is quite low at low airspeeds e.g., in the clean configuration at sea level at 90 mph, it is approximately 65% compared to 81% at typical higher cruise speeds.

The maximum BHP available is assumed to be: at sea level 278 BHP, at 5000 ft 242 BHP, and at 10,000 ft 208 BHP as indicated in the graph titles.[‡] The intersections of the THP_{avl} and the THP_{req} curves represent possible

[*] The values of f shown in the legend were determined by actual flight tests on an E33A Bonanza.

[†] Propeller efficiency is derived from Fig 3-20 in Perkins, C.D. & Hage, R.E., *Airplane Performance Stability and Control*, John Wiley & Sons, based on an 80 inch diameter 3-blade propeller with corrections.

[‡] As determined from the Continental altitude performance chart for an IO-520BB with corrections.

steady level flight conditions at maximum available thrust horsepower. For example: At sea level in the clean configuration the THP_{avl} and THP_{req} curves intersect at approximately 212 mph, which correlates quite well with the 208 mph given by Larry Ball.*

The equivalent airspeed (EAS) for maximum lift to drag ratio, $EAS_{L/D\mathrm{max}}$, which is also the speed for best range and best glide distance, is shown as a dashed line on the graphs. $EAS_{L/D\mathrm{max}}$ decreases as configuration changes increase the parasite drag contribution, f. As the equation below shows, the equivalent airspeed, $EAS_{L/D\mathrm{max}}$, decreases inversely as the fourth root of f, everything else being equal

$$(EAS_{L/D\mathrm{max}})^2 = \frac{2n}{\rho_{\mathrm{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}} = \text{Constant}\,\frac{1}{\sqrt{f}} \tag{3}$$

where n is the load factor, W is the weight, ρ_{SSL} is the density at sea level and e is the so-called Oswald aircraft efficiency factor. Notice that the equivalent airspeed is independent of altitude, i.e., there is no σ (density ratio) in the equation. For example: If you look at the dot representing the $V_{L/D\mathrm{max}}$ in the clean configuration (green curve), the dot occurs at the same EAS for the sea level, 5000 ft and 10,000 ft graphs. Also, notice that the EAS for $V_{L/D\mathrm{max}}$ *decreases* as the aircraft parasite drag increases, i.e., as the aircraft is 'dirtied' up.

Notice also that in the clean configuration the $EAS_{L/D\mathrm{max}}$ is approximately 122 mph which correlates quite well with the value given in the E33A pilot operating handbook (POH).

Also shown on the graphs is the equivalent airspeed for the maximum rate of climb, $EAS_{R/C\mathrm{max}}$ (EAS_y), for the various configurations. The actual rate of climb is proportional to the vertical distance between the THP_{avl} and THP_{req} curves. The greater the distance between the curves, the larger the rate of climb. For example: In the clean configuration at sea level on a standard day the graph shows that the THP_{req} is approximately 78 and the THP_{avl} is approximately 204 at a speed of approximately 119 mph. Recall that rate of climb is given by

$$\mathrm{R/C} = \frac{THP_{\mathrm{avl}} - THP_{\mathrm{req}}}{W} = \frac{(204-78)550}{(3300)60} = 1260\ \text{fpm} \tag{4}$$

which agrees reasonably well with the E33A POH value of 1220 fpm and the value of 1200 fpm given by Ball*. The early E33A POH† gives $EAS_{R/C\mathrm{max}}$ as 112.5 mph. The sea level clean configuration in Chart 1 of 119 mph at 3300 lbs compares reasonably well. The difference is attributed to propeller efficiency for a 3-blade compared to the 2-blade propeller assumed in the early POH.

The graphs show that $EAS_{R/C\mathrm{max}}$ *decreases* with the increasing parasite drag associated with gear and/or flap extension.

The area between the THP_{avl} and the THP_{req} curves can also be thought of as the excess energy available for maneuver. We'll talk about that in Part 2.

Additionally, the graphs show the equivalent airspeed for $EAS_{\gamma\mathrm{max}}$ (EAS_x), i.e., the maximum climb angle. For example: At sea level in the clean configuration $EAS_{\gamma\mathrm{max}}$ is approximately 82 mph. The early POH shows that value to be 91 mph. Again, the difference is attributed to the difference between the efficiency of the three-blade compared to the two-blade propeller assumed in the early POH.

Again, $EAS_{\gamma\mathrm{max}}$ *decreases* for increasing parasite drag, f, associated with gear and/or flap extension. Notice the speeds for $EAS_{\gamma\mathrm{max}}$ and $EAS_{R/C\mathrm{max}}$ are converging with the increasing parasite drag associated with extension of gear and flaps.

Notice THP_{avl} is more than THP_{req} for some range of speed for all three graphs, i.e., at sea level, 5000 ft and 10,000 ft. Hence, the aircraft will climb if flown at the correct speed. But, also notice that the range of speeds where THP_{avl} is more than THP_{req} is significantly reduced with increasing altitude and increasing parasite drag. Furthermore,

*Ball, L., *Those Incomparable Bonanzas*, McCormick-Armstrong, Wichita, KS, 1971.

†Beechcraft Bonanza Model E33A Owners's Manual, Beech Aircraft Corporation, Wichita, KS, 1967.

notice that the available speeds for positive rates of climb as the aircraft configuration becomes 'dirtier' occur with reduced propeller efficiency. The result is reduced thrust horsepower available. Hence, speed control becomes more important.

Sea Level

At sea level only one intersection of the THP_{avl} and THP_{req} curves occurs and occurs at a speed higher than the speed for $V_{P_{R_{\text{min}}}}$, i.e, the speed for minimum power required (maximum endurance). Thus, even at or near stall, excess THP_{avl} is available. Hence, at full power at sea level the aircraft can climb in a near stalled condition, i.e., with the flow separated on a significant portion of the upper wing surface but below the critical angle of attack.

5000 ft

Now, consider the details at 5000 ft altitude. Here, all the THP_{req} curves have moved up and to the left compared to the sea level curves. Notice that there are now two intersections of the THP_{avl} and THP_{req} curves for the gear and flaps 32° configuration compared to only one at sea level. For the gear down and flaps 32° configuration, the low speed intersection occurs at or near stall. Thus, at 5000 ft there is little or no excess power available near stall in that configuration. Hence, the rate of climb is very small or negative. With gear down and flaps 32° the aircraft may not be able to maintain steady level flight near stall. In fact, the aircraft may well descend in that configuration at full power.

The 5000 ft graph also shows that $EAS_{R/C\text{max}}$ and $EAS_{\gamma\text{max}}$ converge towards a single point with the gear extended and the flaps fully extended to 32°. Here, the propeller efficiency is reduced to approximately 60%, which significantly reduces the available thrust horsepower compared to higher speeds.

Notice that in the clean configuration at 5000 ft on a standard day $EAS_{L/D\text{max}}$ is still 122 mph as it was a sea level. This confirms that $EAS_{L/D\text{max}}$ is independent of altitude as shown in Eq. (2). Furthermore, carefully compare the $EAS_{L/D\text{max}}$ values for the various configurations and notice that for all the configurations they are the same as at sea level and hence independent of altitude.

However, comparing the speeds for $EAS_{R/C\text{max}}$ notice that they *decrease* for all configurations. This is because the THP_{req} curves shifts up and to the left and the THP_{avl} curve shifts down and to the left with increasing altitude. Thus, the maximum distance between the THP_{avl} and THP_{req} curves, which represents the maximum rate of climb and the speed for maximum rate of climb, moves to the left and hence to a lower speed. Just by visually comparing the distance between the THP_{req} and the THP_{avl} curves for sea level and 5000 ft it is easily seen that the maximum rate of climb at 5000 ft is also lower than at sea level. The reduction is especially noticeable for the gear and flaps 32° configuration. The propeller efficiency is also somewhat lower.

Looking now at the black dashed line we notice that at 5000 ft $EAS_{\gamma\text{max}}$ increases compared to sea level for all configurations. Furthermore, the maximum climb angle *decreases* compared to those at sea level for all configurations.

Notice that for the clean configuration the THP_{avl} and THP_{req} curves intersect at an EAS of 195 mph. An EAS of 195 mph corresponds to a TAS of 210 mph at 5000 ft which is essentially the same TAS as at sea level.

10,000 ft

Finally consider the details at 10,000 ft altitude. Again, notice that all the thrust horsepower required curves have moved further up and to the left compared to both the sea level and the 5000 ft curves.

For all configurations there are now two intersections of the THP_{avl} and THP_{req} curves. Thus, there are two possible steady level flight speeds – one below the minimum thrust horsepower required point and one above. Recall that below the speed for minimum thrust horsepower required the aircraft does not have speed stability. Hence, flight at this lower speed requires more effort from the pilot.

Here, the speed range for steady level flight is again reduced for all configurations and the propeller efficiency is low in the available speed range.

Notice that the $EAS_{L/D\text{max}}$ is still 122 mph, as it should be, because $EAS_{L/D\text{max}}$ is independent of altitude.

At 10,000 ft there is a significant reduction in positive rate of climb, and the speed range for a positive rate of climb, for the gear down flaps 20° and gear down and flaps 32° configurations. Looking at the 10,000 ft graph the THP_{avl} is approximately 135 THP and the THP_{req} is approximately 115 THP. Hence, the maximum rate of climb with gear and flaps extended to 32° is approximately 200 fpm. The $EAS_{R/C\text{max}}$ is approximately 77 mph EAS , which is considerably slower than the $EAS_{R/C\text{max}}$ of approximately 110 mph EAS in the clean configuration. The speeds for $EAS_{R/C\text{max}}$ for the gear down and gear extended and flap 20° are also lower than that for the clean configuration.

The speeds for $EAS_{\gamma\text{max}}$ for gear down, gear down and flaps 20° and gear down and flaps 32° are also lower than for the clean configurations. Furthermore the rate of climb at the speeds for maximum climb angle are considerably lower than in the clean configuration. For example: with the gear and flaps extended 32° the rate of climb at $EAS_{\gamma\text{max}}$ of approximately 75 mph EAS is 190 fpm. For comparison, at an $EAS_{\gamma\text{max}}$ of approximately 89 mph EAS the rate of climb is approximately 665 fpm in the clean configuration. That is a considerable difference.

The high speed intersection of the THP_{avl} and THP_{req} curves is now at an EAS of approximately 179 mph. The true airspeed is then approximately 208 mph TAS, which is essentially the same as at sea level.

Operational Considerations

Maximum Lift to Drag Ratio

Looking at the graphs clearly shows that the equivalent airspeed for maximum lift to drag ratio, or best glide speed, significantly decreases with configuration changes. For example, the graphs and Eq. (3) above show that although $EAS_{L/D\text{max}}$, for a given weight, does not change with altitude it does decrease significantly with configuration changes. For example, it decreases from approximately 122 mph in the clean configuration, to approximately 96 mph with the gear down, to approximately 91 mph with gear down and 20° of flaps, to approximately 85 mph with the gear down and 32° of flaps. That is a 37 mph difference. Hence, for example, during a glide to an emergency landing with the gear down and flaps up the best glide speed is 26 mph EAS less than with the gear up.

Hot High Altitude Approach

Consider the configuration for a hot high altitude approach to landing, e.g., at the Grand Canyon National Airport (KGCN). At KGCN the runway is 8999 ft long at an elevation of 6609 ft. On a warm summer day at any temperature much above 90°F (ISA +34°C) on the runway the density altitude is over 10,000 ft. At 10,000 ft the graph in Fig. 1 shows that if the aircraft is configured gear down and flaps 32° at 3300 lbs the aircraft has a limited speed range where a positive rate of climb exists, specifically between approximately 56 <= mph EAS <= 99. Furthermore, as mentioned above, the maximum rate of climb is less than 200 fpm. More importantly, the speed for maximum rate of climb is 77 mph EAS, which is 33 mph EAS *less* than the speed for maximum rate of climb at sea level on a standard day at gross weight of 119 mph EAS. This is the value for maximum rate of climb that is typically provided in many pilot operating handbooks. Pilots typically memorize this value. What is more important is that, if an attempt is made to climb with the gear extended and flaps 32° at 119 mph EAS, the aircraft will not climb at a 10,000 ft density altitude. In fact, the 10,000 ft graph shows that, with the gear extended and 32°, the rate of climb is negative at approximately -500 fpm.

Finally, with the gear extended and flaps 32° at a typical approach speed of 100 mph EAS at maximum gross weight, the rate of climb is slightly negative. If a go around is attempted at an EAS of 100 mph, the aircraft will not climb even at full power. If the aircraft is pitched up to increase the angle of attack, which results in a lower airspeed, then the aircraft will, in due course, begin to climb but anemically. If the aircraft continues to pitch up to increase angle of attack, the speed continues to decrease and eventually the rate of climb also decreases until the aircraft either sinks into terrain or departs controlled flight.

How about gear down and flaps 20°? Here, the speed range for a positive rate of climb is expanded to approximately 56 mph to 112mph EAS. The best rate of climb speed, $EAS_{R/C\text{max}}$, is approximately 82 mph EAS , which is considerably lower than the sea level *clean* best rate of climb of 119 mph EAS. However, even at the best rate of climb speed the climb rate is still rather anemic at approximately 300 fpm. In fact, at a typical approach speed of 100 mph the 10,000 ft graph shows that aircraft rate of climb is only 200 fpm. If the aircraft is then pitched up to increase angle of attack and reduce speed, the aircraft may end up on the backside of the power required curve, i.e, below approximately at 70 mph EAS. On the backside of the power required curve the aircraft does not have speed stability. A further increase in angle of attack may result in the aircraft departing from controlled flight.

Consider a landing with gear down and no flaps extended. Here, at 10,000 ft at gross weight the possible range of EAS approach speeds with a positive rate of climb expands to approximately 54 mph to 124 mph EAS with a best rate of climb speed of 110 mph EAS. This is the best option, although the available rate of climb is still only on the order of 400 fpm if a go-around is required.

Hot High Altitude Go-around

The comments above are an excellent preparation for a discussion of a hot high altitude go-around. Specifically, in order to successfully execute a hot high altitude go-around, the approach configuration must be selected with the go-around in mind. In most cases, the proper configuration is gear down and no flaps extended provided the runway is long enough for a successful landing in that configuration.

Even at relatively low field elevations on the mountainous portions of the East Coast, e.g., 1747 ft at Lake Placid, New York (KLKP), runway temperatues much above 100°F yield density altitudes of nearly 5000 ft. Looking at the graph for 5000 ft clearly shows that with gear down and flaps extended the range of speeds for a positive rate of climb is significantly reduced from that at sea level. Notice also that the propeller efficiency is lower, which contributes to the reduced rate of climb. The equivalent airspeed for maximum rate of climb is also decreased compared to that at sea level. In fact, if a go-around is attempted at the sea level speed for maximium rate of climb, i.e., approximately 111 mph, the aircraft will climb, although somewhat anemically. Specifically, the aircraft will climb at approximately 85 mph EAS with gear down and flaps extended 20° at the specified gross weight of 3300 lbs although at an anemic 300 fpm.[*] Again, proper selection of the approach configuration is important in the event of a go-around.

Furthermore, as the density altitude increases, the equivalent airspeeds for maximum rate of climb and maximum climb angle converge or even cross at high density altitudes. Hence, the lowest equivalent airspeed during a go-around is the equivalent airspeed for maximum climb angle. Operating below the equivalent airspeed for maximum climb angle results in a further reduction in rate of climb because of decreasing propeller efficiency.

Hot High Altitude Takeoff

Again, the comments above are an excellent primer for understanding aircraft configuration for a hot high altitude takeoff. Takeoff is complicated by the effects of weight, temperature, runway length and obstacles. In addition, the aircraft may lift off in ground effect but be unable to climb out of ground effect. Figure 1 does not account for ground effect.

The graph in Fig. 1 for 10,000 ft and a weight of 3300 lbs clearly shows that gear down and flap deflection of 20° and 32° result in low climbout rates of climb, e.g., for 20° the maximum rate of climb is about 300 fpm at an $EAS_{R/C\max}$ or about 84 mph EAS. For gear extended and 32° the maximum rate of climb is about 200 fpm at an mph EAS of 77. Furthermore, provided sufficient runway is available and the runway departure end is free of obstacles, Fig. 1 suggests a no flap takeoff. Here, Fig. 1 suggests allowing the aircraft to accelerate in ground effect, raising the gear when a landing on the remaining runway is no longer possible and, if terrain allows, accelerating to $EAS_{R/C\max}$ and then climbing.

The graph for 5000 ft in Fig. 1 illustrates that the equivalent airspeed range that results in a positive rate of climb is significantly expanded. Here, with gear down and flaps 32° the maximum rate of climb is approximately 440 fpm; with gear down and flaps 20° it is approximately 560 fpm, and with gear down and no flaps it is approximately 650 fpm.

[*] Note that the rate of climb approximately increases as the ratio of the gross weight to that of the current weight, i.e., as $W_{gross}/W_{current}$.

Conclusions

Aircraft configuration has a significant effect on the equivalent airspeeds for $EAS_{L/D\max}$, $EAS_{R/C\max}$ and $EAS_{\gamma\max}$. As additional parasite drag is added with the extension of the gear and/or the deflection of flaps, these speeds significantly decrease.

The available speed ranges for positive rates of climb decrease as a result of deploying gear and/or flaps especially at higher equivalent airspeeds.

Using the sea level clean speeds for $EAS_{L/D\max}$, $EAS_{R/C\max}$ and $EAS_{\gamma\max}$ may result in significant reductions in aircraft performance.

Increasing density altitude further exascerbates these effects to the point where the aircraft cannot climb nor in some configurations maintain steady level flight.

At high density altitudes the aircraft configuration for approach and landing should be considered in the light of a possible go around.

For high density altitude takeoffs the aircraft should be configured to minimize parasite drag.

Aircraft configuration may be the root cause of many takeoff and landing loss of control accidents.

Notes

Note 1: Power Required Curve To Maintain Level Flight

$$P_r = \underbrace{\frac{\sigma\rho_{\text{SSL}}}{2} f V^3}_{\text{parasite}} + \underbrace{\frac{2}{\sigma\rho_{\text{SSL}}}\frac{1}{\pi}\frac{1}{e}\left(\frac{W}{b}\right)^2 \frac{1}{V}}_{\text{effective induced}} \tag{5}$$

where

b	is the wing span;
e	is the so called Oswald efficiency factor;
f	is the equivalent flat plate area; or the equivalent parasite drag area;
W	is the weight of the aircraft;
V	is the true airspeed (TAS);
ρ_{SSL}	(rho standard sea level) is the density at sea level;
σ	(sigma) is the ratio of the density at altitude to that at sea level ρ/ρ_{SSL}.

Recall that the Equivalent Airspeed is

$$EAS = \sqrt{\sigma}V = \sqrt{\sigma}TAS \tag{6}$$

where V is synonymous with TAS.

The equivalent power required to maintain level flight can then be rewitten in terms of the EAS as

$$EASP_r = \sqrt{\sigma}P_r = \underbrace{\frac{\rho_{\text{SSL}}}{2} f (EAS)^3}_{\text{equivalent parasite}} + \underbrace{\frac{2}{\rho_{\text{SSL}}}\frac{1}{\pi}\frac{1}{e}\left(\frac{W}{b}\right)^2 \frac{1}{EAS}}_{\text{equivalent effective induced}} \tag{7}$$

Looking at this result carefully, note that for a given aircraft the wing span, b, is typically not going to change nor is the density at sea level on a standard day, ρ_{SSL}. Thus, the power required to maintain level flight changes with equivalent airspeed, EAS, weight, W, the equivalent parasite drag, f, and the Oswald aircraft efficiency factor, e.

Typically the Oswald efficiency factor remains in a narrow band between $0.7 <= e <= 0.8$. Hence, it has only a minor influence on the equivalent induced power required. Thus, for a specific aircraft, the equivalent power required depends principally on the weight, W, the equivalent parasite drag, f, and, of course, the equivalent airspeed, EAS.

Thus, the equivalent parasite power required to maintain level flight is mostly dependent on the equivalent airspeed because the equation contains EAS^3 and on the equivalent parasite drag, f. The principal changes in f are a result of configurations changes, e.g., extension of the gear, flaps, cowl flaps, etc.

The equivalent induced power required to maintain level flight depends on the square of the weight, W^2, and inversely on the equivalent airspeed, EAS, and, to a minor extent, on the Oswald aircraft efficiency, e.

Note 2: Propeller Corrections

Propeller efficiency is derived from Fig 3-20 in Perkins, C.D. & Hage, R.E., *Airplane Performance Stability and Control*, John Wiley & Sons based on an 80 inch diameter 3-blade propeller with a total activity factor of, $AF = 345$ and a power coefficient, $C_PX = 0.1$. The curve for $C_PX = 0.1$ was digitized and a 4th degree polynomial fit to the digitized points. The resulting equation is

$$\text{Efficiency} = \eta = -0.0071378x^4 + 0.088894x^3 - 0.43380x^2 + 0.97850x + 0.006827 \tag{8}$$

where

$$x = \frac{J}{C_p^{(1/3)}} \tag{9}$$

$$C_p = \frac{\text{BHP}}{\sigma \rho_{\text{SSL}} N^3 D^4} \qquad \text{and} \qquad J = \frac{V}{ND} \tag{10}$$

where C_p is the propeller power coefficient, BHP is brake horsepower, N is revolutions per second, D is propeller diameter and J is the advance ratio.

The simplist propeller analysis is simple momentum theory, also known as actuator disk theory. Momentum theory assumes that the propeller disk is replaced by an actuator disk that has in infinite number of blades and that is capable of producing a uniform change in velocity of the airstream passing through the disk. The next levels of detail in propeller theory are blade element theory and vortex theory. (For details see Dommasch, Sherby, and Connolly, *Airplane Aerodynamics* Pitman, New York, 1967)

Basically, propeller efficiency is dependent of the propeller disk area. The larger the propeller disk area the more efficient the propeller is within limits. A simple correction to propeller efficiency based on the momentum theory is to reduce the propeller disk area by the area of the spinner disk, including the non-performing cylindical area of the propeller blades where the blades enter the spinner.

For an E33A Bonanza, the base of the spinner is 13.75 inches in diameter. Inspection of an 80 inch diamemter 3-bladed propeller suggests that approximately 2 inches of the blade where it enters the spinner is non-performing. Hence, the total diamenter of the non-performing area is 17.75 inches. The non-performing area of the propeller disk is thus 247.45 in^2. The disk area for an 80 inch diameter propeller is 5026.55 in^2. Thus, the active propeller disk area is 95.0771% of the actual area. Hence, the calculated propeller efficiency is reduced by multipling it by 0.950771. This is why the maximum propeller efficiency in Fig. 1 is 81%.

Note 3: Brake Horsepower Available

Brake horsepower available is determined from the IO-520BB Altitude Performance Curve. (See Operation and Installation Manual, Models IO-520-B,BA,BB,C,CB,M & MB, Teledyne Continental Motors, FORM X30618, Mar 1994.)

The sea level brake horsepower available is based on flight tests showing a reduction in manifold pressure of 0.8"Hg because of an installed Brackett air filter, i.e., for 28"Hg MP rather than 28.8"MP. The brake horsepower available at 5000 ft and 10,000 ft is *not* reduced because flight test data for MP at those altitudes is unavailable.

Appendix A Effect of Weight and BHP – IO-470N (260 BHP) and IO-470J (225 BHP)

There are a number of Bonanzas that are equipped with either the IO-470N (260 BHP) or the IO-470J (225 BHP) engines. Generally, these models have lower gross weights as well. Figure 2 is for a gross weight of 3050 lbsand an IO-470N at 260 BHP at 2625 RPM. The airframe characteristics are assumed to be the same as those for the E33A. Airframe differences that I know about are: V-tail, with or without cowl flaps and without the speed slope windshield compared to the subject E33A. Fundamentally, these differences are not going to change the results and comments significantly for the IO-470N aircraft (see Fig. 2). Generally, the maximum speed is a bit lower as is the rate of climb in the case of the 470N.

With the 470J the difference is more significant. Specifically, the results for the IO-470J at 10,000 ft, especially with the gear and flaps extended, should be paid attention to (see Fig. 3).

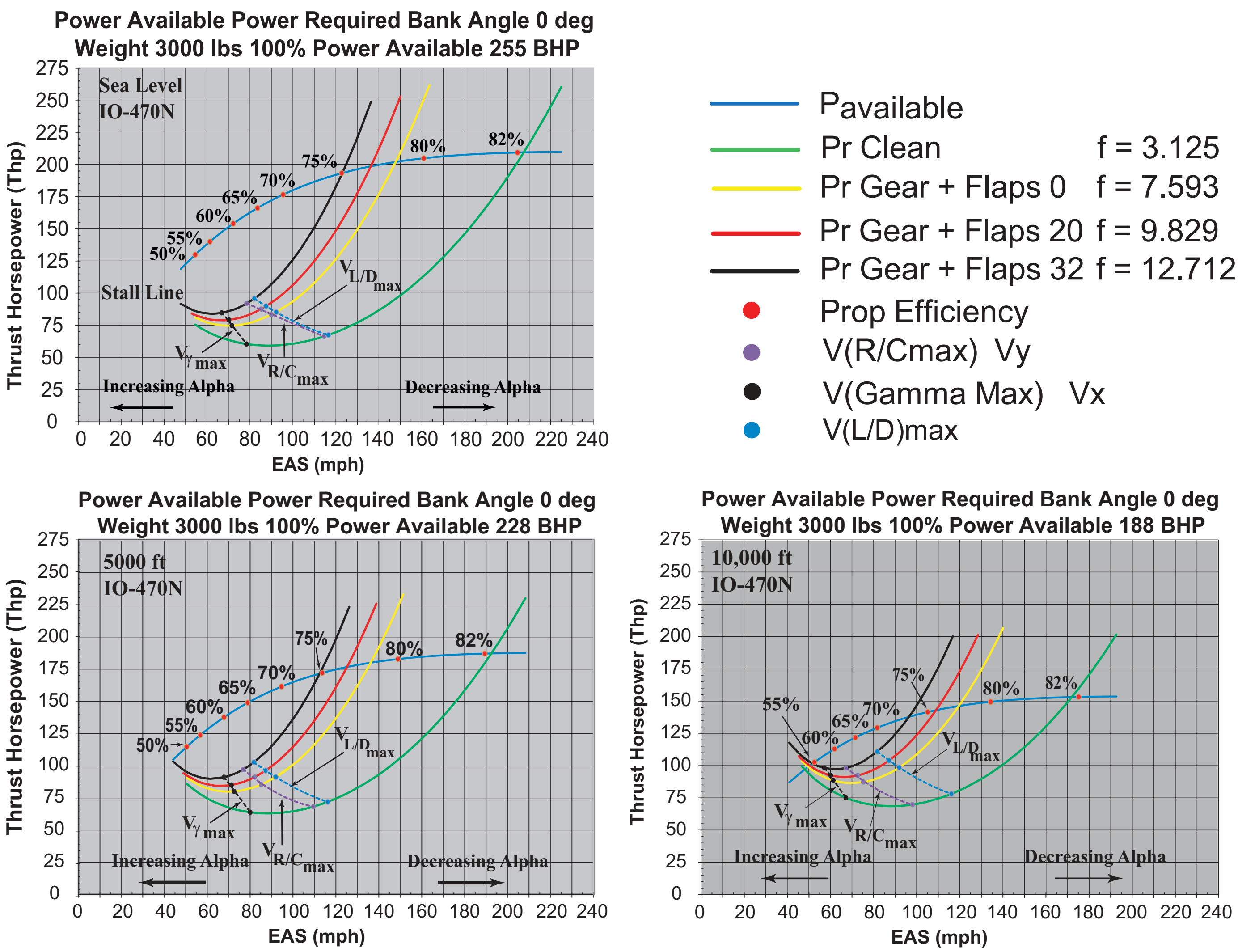

Figure 2. Thrust horsepower available and thrust horsepower required for a normally aspirated aircraft in steady flight without a heading change – **IO-470N**.

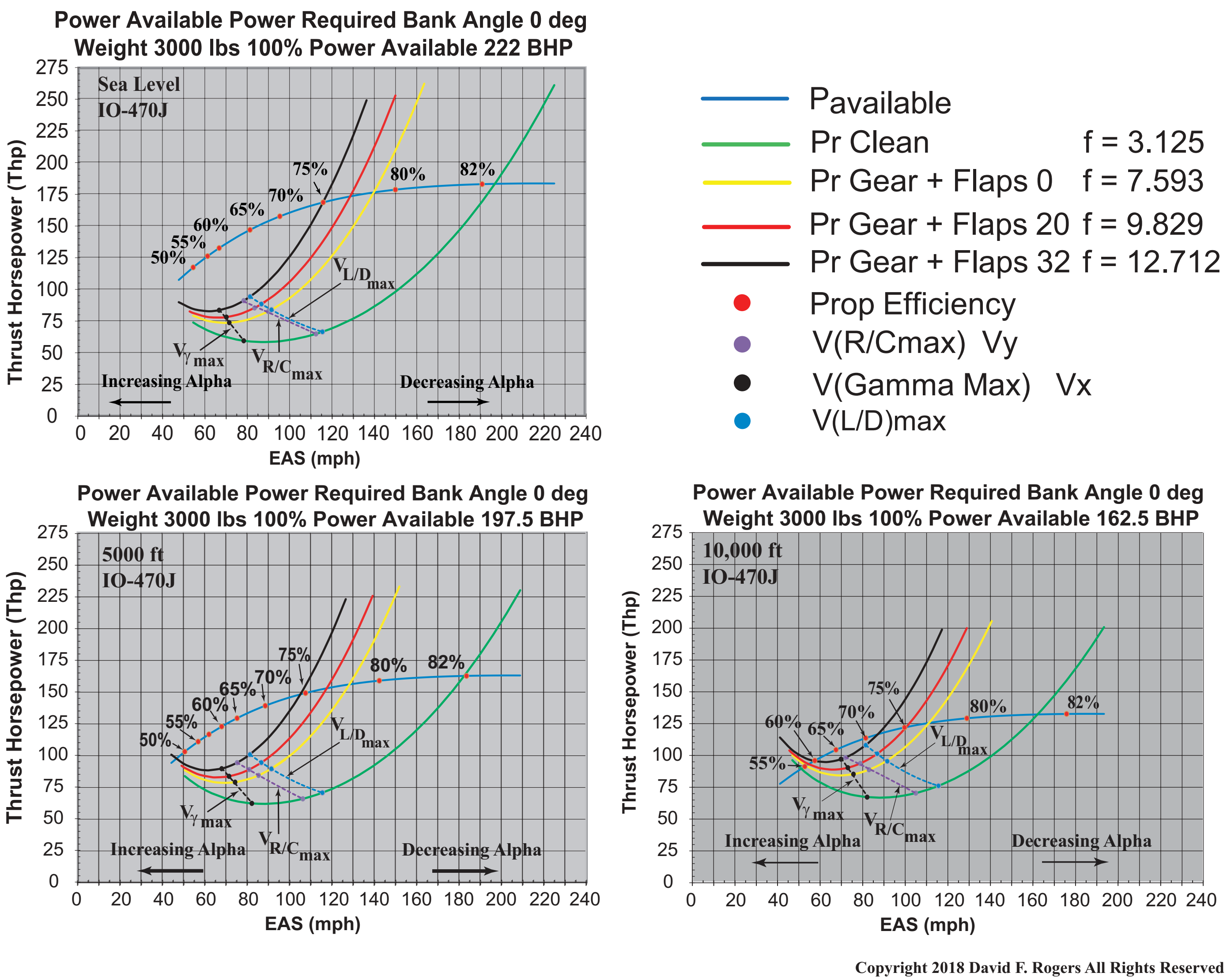

Figure 3. Thrust horsepower available and thrust horsepower required for a normally aspirated aircraft in steady flight without a heading change. – **IO-470J**