Fundamental Angle of Attack

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Is there a fundamental angle of attack for an aircraft? Yes. To begin to understand this consider Fig. 1. Figure 1 shows curves of the power required to maintain level flight versus true airspeed for various altitudes on a standard day. You remember these from the discussion in ground school, don't you? Specifically remember that the power required is composed of two parts, i.e., that due to parasite drag and that due to induced drag. The sum of these results is the hook, or J, shaped curve shown in Fig. 1. Clearly the power required changes with altitude.

You also should remember that a line drawn though the origin (0,0) tangent to, i.e., just touching, the power required curve yields the velocity for maximum lift to drag ratio (best range or best glide) as shown by the dashed line in Fig. 1. Notice that the single dashed line touches every single one of the power required curves for the various altitudes. One could say that the power required curve *slides* along the line for maximum lift to drag ratio with increaseing altitude. Hence, the true airspeed increases with increasing altitude.

However, the angle between the line for maximum lift to drag ratio remains the same, i.e., there is only one dashed line for all altitudes. The angle between the dashed line and the horizontal axis is directly related to the absolute angle of attack for maximum lift to drag ratio. Thus, the angle of attack for maximum lift to drag ratio *does not depend on density altitude.*

Nondimensionalization

Because aerodynamics is a bit complex, aeronautical engineers attempt to simplify problems by combining the problem variables such that the dimensions cancel out, i.e., form a dimensionless or nondimensional variable. Lift coefficient is an example. Figure 1 suggests that the conditions for maximum lift to drag might work. Let's try nondimensionalizing the velocity with the velocity for maximum lift to drag ratio and the power required with the power required for the maximum lift to drag ratio. The result is shown in Fig. 2. It is much more succesfull than one might expect. Not only does nondimensionalization collapse the effect of density altitude, σ (sigma), on the power required, but it also collapses the effects of weight, W, load factor, n, and aircraft configuration, i.e., e and f, into a single curve. The formulas for the velocity and power required for maximum lift to drag ratio are given in the figure. If you can calculate, or find, the velocity and power required for maximum lift to drag ratio, then you can determine the power required for any altitude, any weight, any load factor, and any configuration from Fig. 2 for any velocity. More on that in due course.

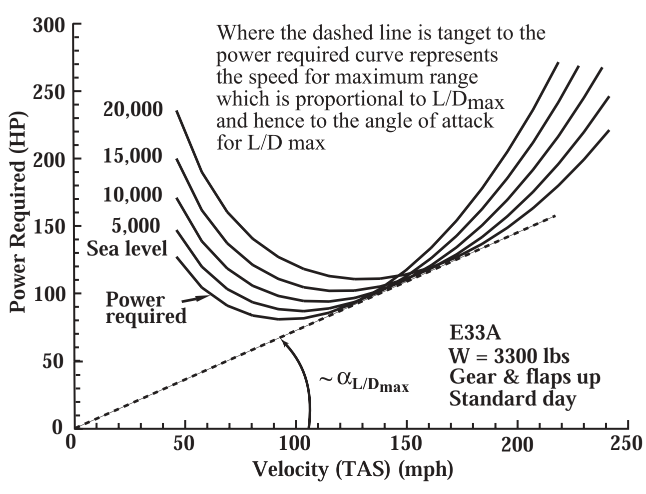

Figure 1. Altitude effect on power required.

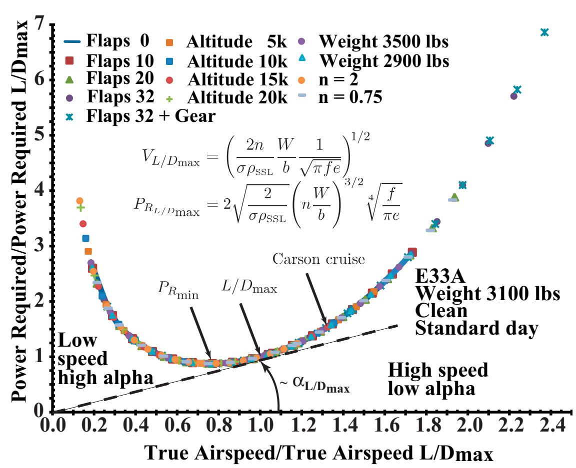

Figure 2. Nondimensional Power Required–Multiple Effects.

Relation Between Angle of Attack and Velocity

So far the discussion has been couched in terms of velocity. Let's now look at the absolute angle of attack. The absolute angle of attack varies inversely with the square of the velocity, i.e., from

$$C_L = a\alpha = \frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2} \quad \text{we have} \quad \alpha = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2} \propto \frac{1}{(V^2)}$$

where a is the lift curve slope (see the Absolute Angle of Attack article).

Figure 3 illustrates the relationship between absolute angle of attack and velocity nondimensionalized with angle of attack and velocity for maximum lift to drag ratio, respectively. The result is a parabolic (second degree) curve. Notice that the angle of attack *decreases* significantly with *increasing* velocity. Conversely, the angle of attack *increases* significantly with *decreasing* velocity. Again, the formulas for absolute angle of attack and power required for maximum lift to drag ratio are given in Fig. 3. Notice that again the results for all the various configurations collapse into a single curve.

Turning now to the small table of equivalent parasite drag values, f, absolute angle of attack and the velocity for maximum lift to drag ratio given in Fig. 3, notice that increasing parasite drag has significant effects. For example, going from a clean configuration to a dirty configuration, e.g., gear down, flaps full and cowl flaps open, decreases the velocity for maximum lift to drag ratio for a typical Bonanza, from 107.8 KTAS to 75.9 KTAS: a significant 29.8%. In fact, the velocity for maximum lift to drag ratio decreases as the fourth root of the ratio of equivalent parasite drag areas ($3.125/12.712 = 0.2458$), as can be seen from the equation above. Taking the fourth root of 0.2458 by taking the square root twice yields $0.704 \times 107.8 = 75.9$ KTAS, which agrees quite nicely with the result in Fig. 3.

However, as also shown in the table in Fig. 3, increasing the parasite drag increases the absolute angle of attack for maximum lift to drag ratio. Using the same example increases the absolute angle of attack from 5.23° to 10.55°. The absolute angle of attack is doubled. In fact, the absolute angle of attack for maximum lift to drag ratio increases as the square root of the equivalent parasite drag, f. Specifically, ($12.172/3.125 = 3.895$). The $\sqrt{3.895} = 1.97$ and $1.97 \times 5.23 = 10.32$, which is close enough to doubling. The Oswald aircraft efficiency factor, $e = 0.56$, was used in calculating all the configurations in Fig. 3.

Fundamental Angle of Attack — $\alpha_{L/D\text{max}}$

From the above, it is clear that the absolute angle of attack for maximum lift to drag ratio is a fundamental aircraft angle of attack. Examining the absolute angle of attack more closely, it is easy* to show that it is independent of density altitude and aircraft weight. Specifically,

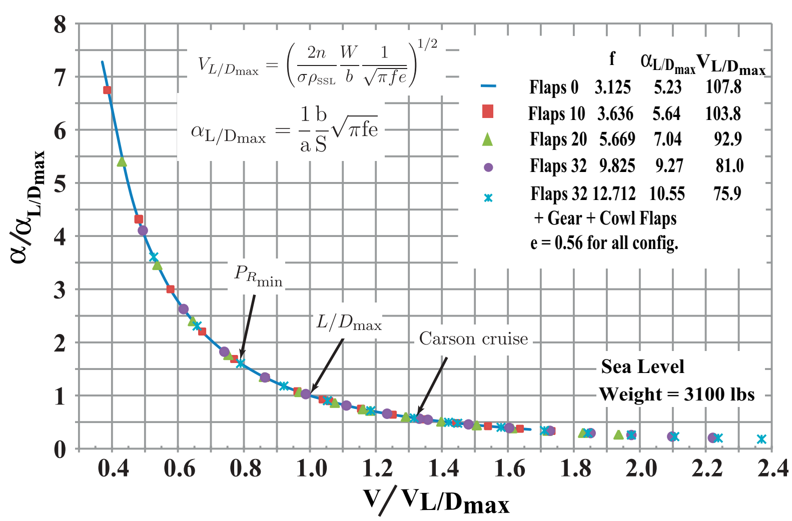

Figure 3. Relationship between nondimensional velocity and nondimensional angle of attack.

*If you are mathematically inclined, the equations in Appendix A will convince you that this result is not happenstance.

$$\alpha_{L/D\max} = \frac{1}{a}\frac{b}{S}\sqrt{\pi f e}$$

Which *does not* depend on weight (W), load factor (n) or density altitude (σ.) The absolute angle of attack for $L/D_{\max}$ depends *only* on aircraft design parameters: wing span (b), wing area (S), parasite drag (f) and Oswald efficiency factor (e). Although the parasite drag, f, and Oswald efficiency factor, e, can be changed in flight, e.g., by extending gear and/or flaps or opening cowl flaps, they are still aircraft design parameters.

In addition, there are two other angles of attack that are simple numerical multiples of the absolute angle of attack for maximum lift to drag ratio, specifically the absolute angle of attack for minimum power required (maximum endurance) and the absolute angle of attack for Carson Cruise. The absolute angle of attack for minimum power required is simply $1.73\alpha_{L/D\max}$ and the absolute angle of attack for Carson Cruise is $0.58\alpha_{L/D\max}$ as also shown in Fig. 3. These angles of attack are also independent of density altitude, weight and load factor. Specifically, the corresponding angles of attack for $P_{R_{\min}}$ and Carson Cruise are

$$\alpha_{P_{R_{\min}}} = \sqrt{3}\,\alpha_{L/D\max} = 1.73\alpha_{L/D\max} \qquad \text{and} \qquad \alpha_{CC} = \frac{1}{\sqrt{3}}\,\alpha_{L/D\max} = 0.58\alpha_{L/D\max}$$

For a typical Bonanza in the clean configuration the angles are:

$$\alpha_{L/D\max} = 5.23^\circ \qquad \alpha_{P_{R_{\min}}} = 9.06^\circ \qquad \alpha_{CC} = 3.02^\circ$$

The velocities for $V_{P_{R_{\min}}}$ and V_{CC} are also multiples of the $V_{L/D\max}$. Specifically,

$$V_{P_{R_{\min}}} = \frac{1}{\sqrt[4]{3}}\,V_{L/D\max} = 0.76V_{L/D\max} \quad \text{and} \quad V_{CC} = \sqrt[4]{3}\,V_{L/D\max} = 1.32V_{L/D\max}$$

However, the velocity (TAS) for $V_{L/D\max}$ does depend on weight, density altitude and load factor as shown by the equation in Fig. 3. Hence, $V_{P_{R_{\min}}}$ and V_{CC} also depend on weight, density altitude and load factor.

Power Required and Angle of Attack

Recalling the inverse square relationship between angle of attack and velocity allows recasting the nondimensional power required in terms of the angle of attack. However, it is convenient to recast it in terms of the nondimensional square root of the absolute angle of attack over the angle of attack, as shown in Fig. 4. Here we see that Fig. 4 looks very similar to Fig. 2. Specifically, notice that again a line through the origin tangent to the curve is related to the absolute angle of attack for maximum lift to drag ratio. That is why it was recast in terms of the square root of the absolute angle of attack for maximum lift to drag ratio divided by the absolute angle of attack.

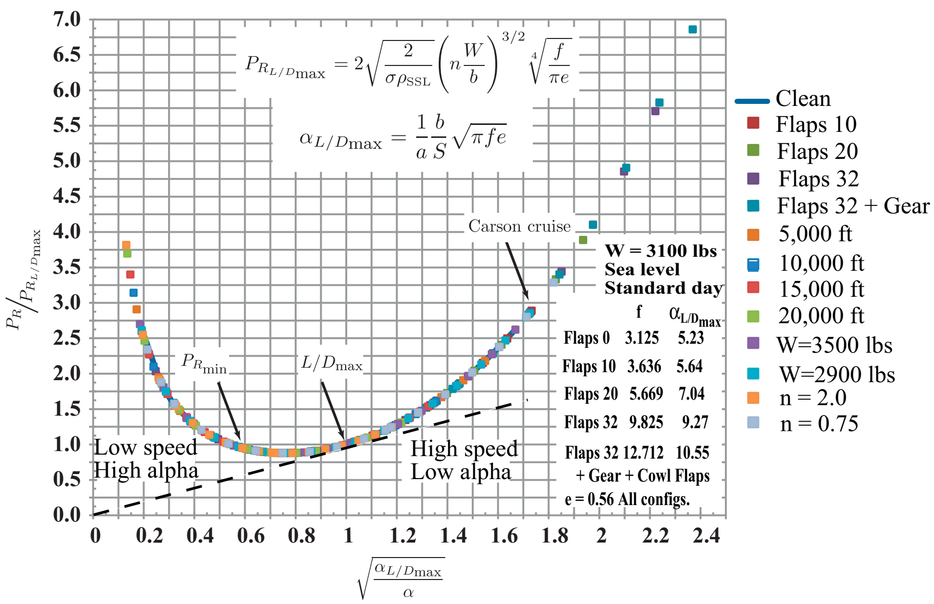

Figure 4. Relationship between nondimensional power required and nondimensional angle of attack.

Summary

Four absolute angles of attack that are independent of density altitude and weight and only depend upon aircraft design parameters, specifically the absolute angle of attack for maximum lift to drag ratio, the absolute angle of attack for minimum power required and the absolute angle of attack for Carson Cruise as well as the stall angle of attack, have been identified.

The angle of attack for maximum lift to drag ratio is identified as a fundamental aircraft angle of attack.

Using the conditions for maximum lift to drag ratio to nondimensionalize the power required, angle of attack and velocity collapse the curves into a single curve for density altitude, weight, load factor and configuration changes, i.e., e and f.

The equivalent parasite drag area, f, has significant effects on the velocity and absolute angle of attack for maximum lift to drag ratio. The velocity for minimum power required and Carson cruise are also similarly affected. Specifically, increasing values of equivalent parasite drag area, e.g., extending gear, full flaps and opening cowl flaps decreases, the velocity for maximum lift to drag ratio, minimum power required and Carson cruise by 29% for a typical Bonanza. The angle of attack is doubled.

Recasting the nondimensional power required curve in terms of the square root of angle of attack divided by the angle of attack for maximum lift to drag ratio presents the results in a form equivalent to the classical power required versus velocity curve. A tangent line through the origin again yields the conditions for maximum lift to drag ratio.

Appendix A

Fundamental Angle of Attack

From the definition of lift coefficent

$$C_L = a\alpha = \frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2}$$

or

$$\alpha = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2}$$

At L/D_{max} the angle of attack is

$$\alpha_{L/D\text{max}} = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2_{L/D\text{max}}}$$

The velocity for $V_{L/D\text{max}}$ is

$$V_{L/D\text{max}} = \left(\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}\right)^{1/2}$$

or

$$V^2_{L/D\text{max}} = \frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{b}\frac{1}{\sqrt{\pi f e}}$$

Substituting into the equation for $\alpha_{L/D\text{max}}$

$$\alpha_{L/D\text{max}} = \frac{1}{a}\frac{2n}{\sigma\rho_{\text{SSL}}}\frac{W}{S} \quad \frac{\sigma\rho_{\text{SSL}}}{2n}\frac{b}{W}\frac{\sqrt{\pi f e}}{1}$$

or

$$\alpha_{L/D\text{max}} = \frac{1}{a}\frac{b}{S}\sqrt{\pi f e}$$

which does not depend on weight (W), load factor (n) or density altitude (σ) but, only on aircraft design parameters: wing span (b), wing area (S), parasite drag (f) and Oswald efficiency factor (e).