An Alternate Technique for Determining the Equivalent Flat Plate Area by David F. Rogers

With the advent of GPS and the Horseshoe Heading technique for directly determining true airspeed, an alternate technique for determining the equivalent flat plate area, f, and the Oswald or airplane efficiency factor using true airspeed rather than the calibrated airspeed is of interest.

Recall that the thrust horsepower required, THP_r is given by

$$THP_r = \underbrace{\frac{\sigma\rho_{\text{SL}}}{2} f V^3}_{\text{parasite}} + \underbrace{\frac{2}{\sigma\rho_{\text{SL}}} \frac{1}{\pi e} \left(\frac{W}{b}\right)^2 \frac{1}{V}}_{\text{effective induced}}$$

where

- b is the wing span;
- e is the so called Oswald efficiency factor.
- f is the equivalent flat plate area or the equivalent parasite drag area.
- W is the weight of the aircraft.
- V is the true airspeed (TAS).
- ρ_{SL} (rho sea level) is the density at sea level.
- σ (sigma) is the ratio of the density at altitude to that at sea level ρ/ρ_{SL}.

Multiplying by the true airspeed, V yields

$$THP_r V = \frac{\sigma\rho_{\text{SL}}}{2} f V^4 + \frac{2}{\sigma\rho_{\text{SL}}} \frac{1}{\pi e} \left(\frac{W}{b}\right)^2 = A + BV^4$$

which is a linear relation in V^4 with

$$A = \frac{2}{\sigma\rho_{\text{SL}}} \frac{1}{\pi e} \left(\frac{W}{b}\right)^2 \quad \text{and} \quad B = \frac{\sigma\rho_{\text{SL}}}{2} f$$

Hence, e, the Oswald efficiency is given by the intercept of a straight line on a $THP_r V$ versus V^4 plot and f, the equivalent flat plate area is given by the slope of the straight line. Specifically

$$e = \frac{2}{\sigma\rho_{\text{SL}}} \frac{1}{\pi A} \left(\frac{W}{b}\right)^2 \quad \text{and} \quad f = \frac{2}{\sigma\rho_{\text{SL}}} B$$

Reduction to standard sea level and weight conditions

For any given standard weight, W_{std}, at sea level on a standard day the true airspeed, V_{std}, is given by

$$V_{\text{std}} = \sqrt{\frac{2}{\rho_{\text{SL}}}\frac{W_{\text{std}}}{S}\frac{1}{C_L}}$$

where, S, is the wing planform area.

Similarly, the thrust horsepower required at standard weight at sea level on a standard day, $THP_{r_{\text{std}}}$, is given by

$$THP_{r_{\text{std}}} = \sqrt{\frac{2}{\rho_{\text{SL}}}\frac{W_{\text{std}}^3}{S}\frac{C_D^2}{C_L^3}}$$

At any other conditions the true airspeed, V, and the thrust horsepower required, THP_r, are given by

$$V = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_L}}$$

and

$$THP_r = \sqrt{\frac{2}{\rho}\frac{W^3}{S}\frac{C_D^2}{C_L^3}}$$

At the same lift coefficient, C_L

$$\frac{V_{\text{std}}}{V} = \sqrt{\frac{\rho}{\rho_{\text{SL}}}\frac{W_{\text{std}}}{W}}$$

and

$$\frac{THP_{r_{\text{std}}}}{THP_r} = \sqrt{\frac{\rho}{\rho_{\text{SL}}}\frac{W_{\text{std}}^3}{W^3}}$$

Hence, test conditions can be reduced to standard weight at sea level on a standard day using

$$V_{\text{std}} = V\sqrt{\sigma}\sqrt{\frac{W_{\text{std}}}{W}}$$

and

$$THP_{r_{\text{std}}} = THP_r\sqrt{\sigma}\left(\frac{W_{\text{std}}}{W}\right)^{3/2}$$

Figure 1 shows an example for an model E33A Bonanza for data taken at a test weight of 3140 lbs at a pressure altitude of 6000 feet and reduced to a standard with of 3300 lbs and sea level conditions on a standard day. From the linear regression for the data of Figure 1 without vortex generators fitted

$$e = \frac{2}{(0.8)(0.002378)} \frac{1}{\pi(7.295)} \left(\frac{3140}{33.5}\right)^2 \times 10^{-6} = 0.403$$

and

$$f = \frac{2}{(0.8)(0.002378)} 0.003097 = 3.26\,\text{ft}^2$$

Similarly using the results from Figure 1 with vortex generators fitted yields

$$e = \frac{2}{(0.8)(0.002378)} \frac{1}{\pi(7.3399)} \left(\frac{3140}{33.5}\right)^2 \times 10^{-6} = 0.401$$

and

$$f = \frac{2}{(0.8)(0.002378)} 0.003323 = 3.49\,\text{ft}^2$$

Here, we note that adding vortex generators adds 0.22 ft^2 to the equivalent flat plate area but has little effect of the Oswald efficiency factor. However, the Oswald efficiency factors derived from the data are lower than expected. The lower Oswald efficiency factors are attributed to the fact that all of the data were taken on the 'front side' of the power required curve.

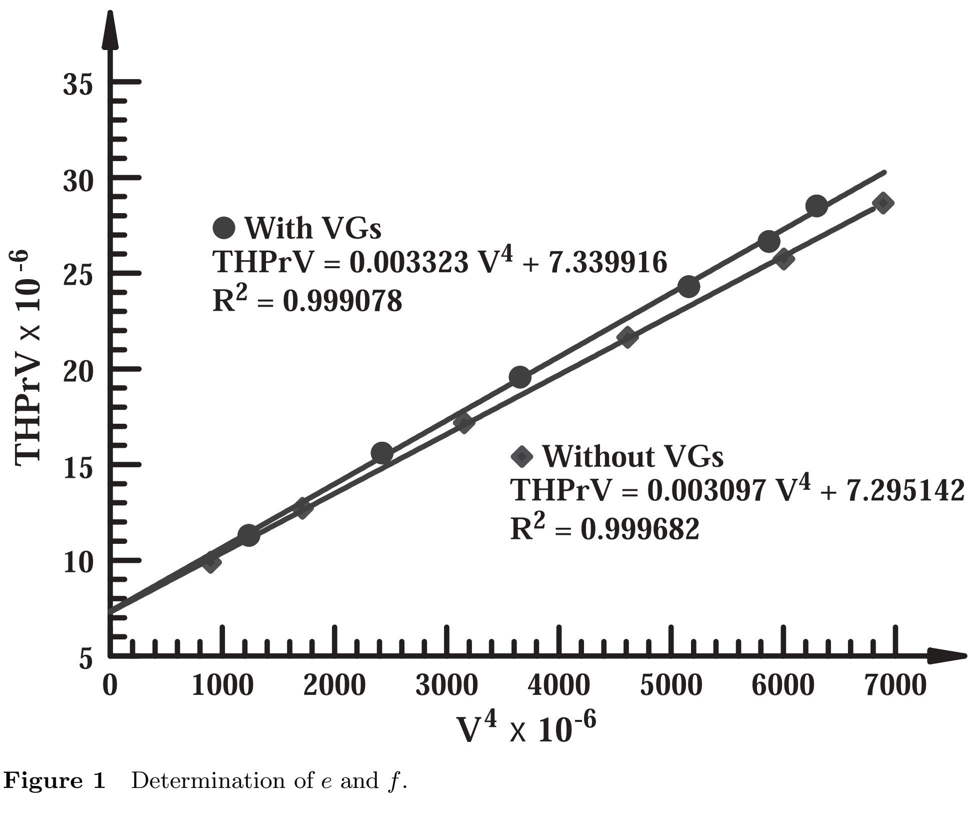

Figure 1 Determination of e and f.

