Estimating The Angle of Attack Cushion

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Introduction

In considering the turnback maneuver there is considerable concern about stalling the aircraft in the optimal 45° banked turn close to the optimal stall speed. Consequently, it is desirable to consider margins. Because the aircraft stalls at the critical angle of attack it is useful to consider margins in terms of angle of attack rather than in terms of speeds. This is relatively easily accomplished as shown below.

Analysis

Consider that both speed and lift coefficient are surrogates for the angle of attack. Thus, consider

$$C_L = a\alpha = \frac{2W}{\sigma\rho_{\text{SSL}}V^2S} \quad \Rightarrow \quad \alpha = \frac{2}{a}\frac{1}{\sigma\rho_{\text{SSL}}}\frac{W}{S}\frac{1}{V^2} \tag{1}$$

Here, W, is the aircraft weight, S, is the aircraft wing area, ρ_{SSL}, is the air density at sea level on a standard day and σ is the density ratio, ρ/ρ_{SSL}. Everything else being equal we have

$$\frac{\alpha}{\alpha_{stall}} = \left(\frac{V_{stall}}{V}\right)^2 \quad \Rightarrow \quad \alpha = \left(\frac{V_{stall}}{V}\right)^2 \alpha_{stall} \tag{2}$$

This result shows that the ratio of any given angle of attack, α, to the stall angle of attack, α_{stall}, is *inversely* related to the *square* of the ratio of the true airspeed, V, to the stall true airspeed, V_{stall}, as indicated in Eq.(2) above. From Eq.(1) above

$$\alpha = \frac{C_L}{a} \tag{3}$$

Equation (3) shows that if the stall lift coefficient, $C_{L_{max}}$, is known, e.g., from the stall speed in the Pilot Operating Handbook (POH), and estimating the aircraft lift curve slope, a, then from Eq.(3) the stall angle of attack can be estimated.

Let's consider three aircraft as examples: a C172M, an E33A Bonanza and an A36 Bonanza.

Examples

If an aircraft has a climb to glide ratio of *less* than one, then it is less likely to be able to return to the runway than an aircraft with a climb to glide ratio *more* than one [1]. A Cessna C172M with a 150 bhp engine has a climb to glide ratio *less* than one. Let's look at that aircraft.

C172M

Based on information from the POH the Cessna 172M has a $C_{L_{max}} = 1.591$ and an equivalent stall airspeed of 57 mph or 83.79 fps. Estimating, a, the lift curve slope for the C172M as 0.078/deg yields

$$\alpha_{stall_{C172}} = \frac{1.591}{0.078} = 20.34^\circ \tag{4}$$

for the absolute angle of attack.

If the C172M, at a gross weight of 2300 lbs on a sea level standard no wind day attempts a turnback maneuver at 5% above the stall speed, i.e., 59.9 mph or 88.0 fps , then, from Eq.(2), the angle of attack during the turn is

$$\alpha_{1.05_{C172M}} = \left(\frac{V_{stall}}{V_{1.05}}\right)^2 \alpha_{stall_{C172M}} = \left(\frac{57}{59.9}\right)^2 \alpha_{stall_{C172M}} = (0.91)(20.34) = 18.44^\circ \tag{5}$$

Here, for the C172M the angle of attack cushion is

$$\alpha_{1.05_{cush}} = 20.34 - 18.44 = 1.9^\circ \tag{6}$$

Similarly, the angle of attack cushions for 10%, 15% and 20% cushions for the C172M are

$$\begin{aligned} \alpha_{1.10_{cush}} &= 20.34 - 16.80 = 3.5^\circ \\ \alpha_{1.15_{cush}} &= 20.34 - 15.42 = 4.97^\circ \\ \alpha_{1.20_{cush}} &= 20.34 - 14.12 = 6.22^\circ \end{aligned} \tag{7}$$

Using the level flight turn radius as an approximation for the descending turnback turn, the increase in turn radius, R, for the C172 for a $\phi = 45^\circ$ bank angle is

$$\begin{aligned} R_{\alpha_{stall}} &= 218\,\text{ft} \\ R_{\alpha_{1.05_{cush}}} &= 240\,\text{ft} \\ R_{\alpha_{1.10_{cush}}} &= 264\,\text{ft} \\ R_{\alpha_{1.15_{cush}}} &= 289\,\text{ft} \\ R_{\alpha_{1.20_{cush}}} &= 314\,\text{ft} \end{aligned} \tag{8}$$

Now let's take a look at an aircraft with a climb to glide ratio of more than one, i.e., an E33A.

E33A

From the POH the stall equivalent airspeed for an E33A is 72 mph or 105.84 fps . Hence, from Eq.(2), the maximum lift coefficient, $C_{L_{max}}$, for the E33A at gross weight is 1.376. Again, estimating, a, the lift curve slope for an E33A as 0.078/deg and using Eq.(3) yields

$$\alpha_{stall_{E33A}} = \frac{1.376}{0.078} = 17.64^\circ \tag{9}$$

for the absolute angle of attack.

If an E33A, at a gross weight of 3300 lbs on a sea level standard day attempts a turnback maneuver at 5% above the stall speed, i.e., 76.8 mph or 111.1 fps , then the angle of attack during the turn is

$$\alpha_{1.05_{E33A}} = \left(\frac{V_{stall}}{V_{1.05}}\right)^2 \alpha_{stall_{E33A}} = \left(\frac{105.84}{111.1}\right)^2 \alpha_{stall_{E33A}} = (0.91)(17.64) = 16^\circ \tag{10}$$

Hence, the angle of attack cushion is

$$\alpha_{1.05_{cush}} = \alpha_{stall} - \alpha_{1.05} = 17.64 - 16 = 1.64^\circ \tag{11}$$

Notice that the angle of attack cushion for the E33A is a little less, 1.64° compared to 1.9°, than for the C172M.

Similarly, the angle of attack cushions for 10%, 15% and 20% above the stall speed for the E33A are

$$\begin{aligned}\alpha_{1.10_{cush}} &= 17.64 - 14.57 = 3.07^\circ \\ \alpha_{1.15_{cush}} &= 17.64 - 12.34 = 4.30^\circ \\ \alpha_{1.20_{cush}} &= 17.64 - 12.24 = 5.40^\circ \end{aligned} \tag{12}$$

Again, using the level turn radius, $R = V^2/(\text{g} \tan(\phi))$, as an approximation for the descending turnback turn for a 45° bank angle, the results are

$$\begin{aligned}R_{\alpha_{stall}} &= 348\,\text{ft} \\ R_{\alpha_{1.05_{cush}}} &= 383\,\text{ft} \\ R_{\alpha_{1.10_{cush}}} &= 421\,\text{ft} \\ R_{\alpha_{1.15_{cush}}} &= 460\,\text{ft} \\ R_{\alpha_{1.20_{cush}}} &= 501\,\text{ft} \end{aligned} \tag{13}$$

From the above notice that as the gross weight and the equivalent stall airspeed of the aircraft *increased*, i.e., from 2300 lbs and 57 mph for the C172M to 3300 lbs and 72 mph for the E33A the angle of attack cushion *decreases*. What if we increase the gross weight, and hence the equivalent stall airspeed, of the E33A to that for an A36 at a weight of 3600 lbs?

A36

From the POH for the A36 the stall equivalent airspeed is 69 kts (79.4 mph) or 116.6 fps . From Eq.(1) above for an A36 at a weight of 3600 lbs with a wing area of 181 ft^2 the $C_{L_{max}}$ for an A36 is 1.265.

Again, estimating, a, the lift curve slope for an A36 as 0.078/deg yields from Eq.(3)

$$\alpha_{stall_{A36}} = \frac{1.265}{0.078} = 16.22^\circ \tag{14}$$

for the absolute angle of attack.

If an A36, at a gross weight of 3600 lbs on a sea level standard day, attempts a turnback maneuver at 5% above the stall speed, i.e., 71.4 kts or 122.4 fps, then the angle of attack during the turn is

$$\alpha_{1.05_{A36}} = \left(\frac{V_{stall}}{V_{1.05}}\right)^2 \alpha_{stall_{A36}} = \left(\frac{116.6}{122.4}\right)^2 \alpha_{stall_{A36}} = (0.91)(16.22) = 14.71^\circ \tag{15}$$

Hence, the angle of attack cushion is

$$\alpha_{1.05_{cush}} = \alpha_{stall} - \alpha_{1.05} = 16.22 - 14.71 = 1.51^\circ \tag{16}$$

Similarly, the angle of attack cushions for 10%, 15%, 20% and 30% above the stall speed for the A36 are

$$\begin{aligned}\alpha_{1.10_{cush}} &= 16.22 - 13.40 = 2.82^\circ \\ \alpha_{1.15_{cush}} &= 16.22 - 12.26 = 3.96^\circ \\ \alpha_{1.20_{cush}} &= 16.22 - 11.26 = 4.96^\circ \\ \alpha_{1.30_{cush}} &= 16.22 - \ \ 9.60 = 6.62^\circ \end{aligned} \tag{17}$$

From these results we see that as speed in the turnback turn increases the angle of attack cushion increases as expected.

Using the level turn radius, $R = V^2/(\text{g} \tan(\phi))$, as an approximation for the descending turnback turn for a 45° bank angle, the results are

$$\begin{aligned}R_{\alpha_{stall}} &= 421\,\text{ft} \\ R_{\alpha_{1.05_{cush}}} &= 464\,\text{ft} \\ R_{\alpha_{1.10_{cush}}} &= 509\,\text{ft} \end{aligned}$$

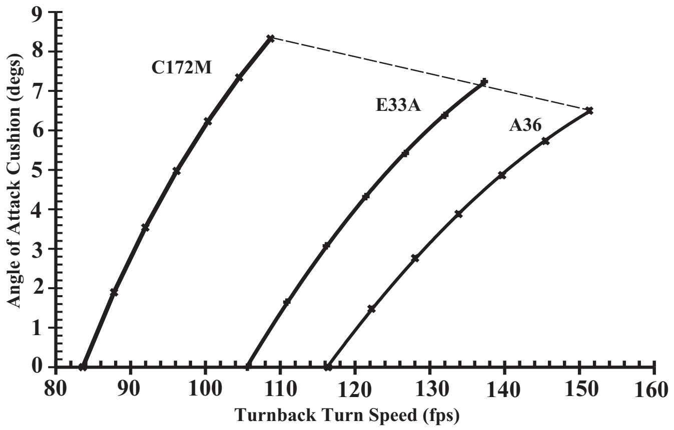

Figure 1. Angle of Attack Cushion vs Turnback Turn Speed In A 45° Bank.

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\begin{aligned}
R_{\alpha_{1.15_{cush}}} &= 556\,\text{ft}\\
R_{\alpha_{1.20_{cush}}} &= 606\,\text{ft}\\
R_{\alpha_{1.30_{cush}}} &= 711\,\text{ft}
\end{aligned}
\tag{18}
$$

Figure 1 compares the results for the C172M, E33A and the A36. As expected, the variation of the angle of attack cushion with turnback speed is parabolic. Figure 1 also shows that the angle of attack cushion *decreases* as the weight, and hence the stall speed, of the aircraft *increases*. For example: for the C172M, the E33A and the A36, the angle of attack cushions at $V_{1.05_{cushion}}$ are 1.9°, 1.6° and 1.5° respectively. In fact, the angle of attack cushion decreases approximately linearly as shown by the dashed line in Fig. 1.[*]

The fundamental reason for this effect is that, for a clean aircraft wing, the angle of attack range is typically between a zero absolute angle of attack on the order of -2° to -4° and a stall absolute angle of attack on the order of 18° to 20° regardless of the aircraft size, weight or operating speed range. In effect, the

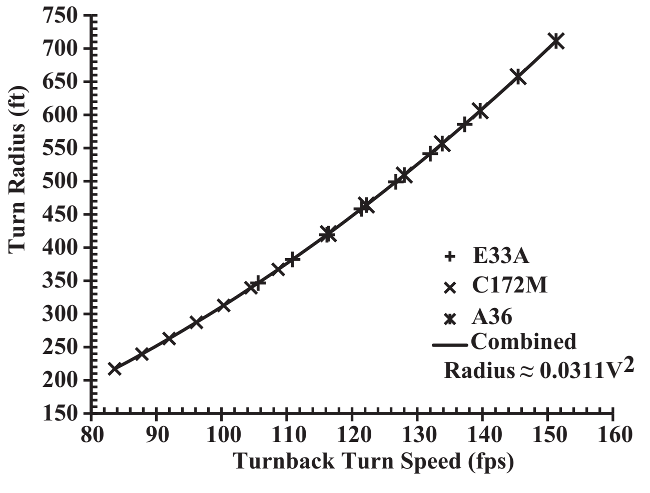

Figure 2. Turn Radius vs Turnback Turn Speed in a 45° Bank.

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[*] The linear variation shown in Fig. 1 by the dashed line is Cushion $= -0.0421\,(1.3V_{stall}) + 12.935$.

absolute angle of attack range for a clean aircraft wing is basically fixed. Hence, as the size, weight and/or operating speed range of the aircraft increases the angle of attack cushion decreases.

Figure 2 shows the effect of turning speed in a banked 45° turn on the radius of the turnback turn. Figure 2 shows the results for all three aircraft from stall speed to 130% of stall speed. Because, for a given bank angle, the turnback turn radius only depends on the square of the turnback speed, the results for all three aircraft fall on a single parabolic curve as also shown in Fig. 2 by the solid line. The result is that the greater the turnback speed, the further from the end of a given runway the aircraft is at the end of the typically 210° turnback turn. Hence, the greater distance the aircraft must glide at $V_{L/D_{max}}$ to reach the runway.

From Figs. 1 and 2 and the above angle of attack cushion and radius results, it is also clear that there is a decided trade-off between executing the turnback turn at or near the aircraft stall speed and the increase in the distance from the runway at the end of the turn and hence the increase in the required glide distance to the runway.

These two factors, among others, must be weighed when deciding on how to fly the turnback maneuver.

References

[1] Rogers, David, F., Estimating The Turnback Altitude, Sourced from https://www.nar-associates.com/technical-flying/impossible/EstimatingTurnbackAltitude.pdf, March 19, 2022.